# Haute-précision, maîtrise des erreurs et incertitudes en CFD


*presenté par*
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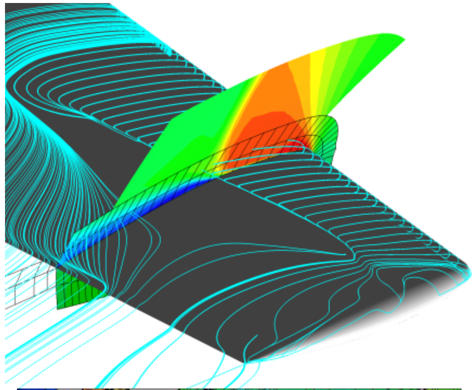
return on innovation

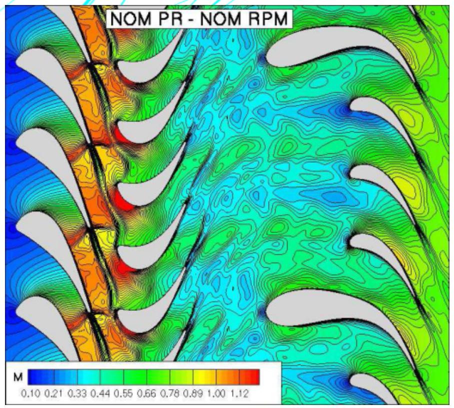
# PLAN

- Intro générale / Challenge en CFD
- Revue des erreurs en CFD
- Haute précision et estimation d'erreur
- Propagation des incertitudes
- Synergie Expérience-Haute précision / Calcul RANS : Modélisation de la turbulence par méthode d'apprentissage

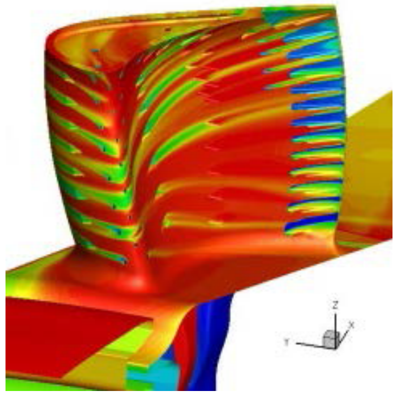
# *Some aspects of ONERA CFD*

- **CFD considered mature technology for nominal flow configurations**
  - Many CFD codes and assessed models are daily used in industry
  - Main CFD codes at ONERA :
    - elsA for aerodynamics, aeroelasticity, aeroacoustics
    - CEDRE for energetics, aeroacoustics, mutiphysics
    - Used by industrial partners Airbus, Safran, EDF, CNES, EDF and ONERA CFD expertise used by Dassault
    - Under development CODA (ONERA, DLR, Airbus), ORION/Mosaic (ONERA, Safran)



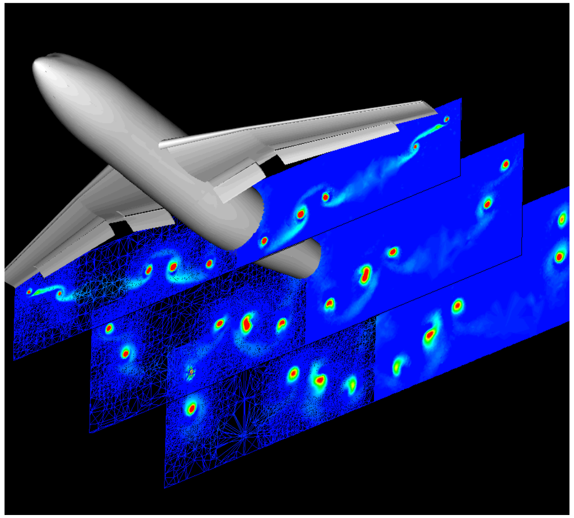


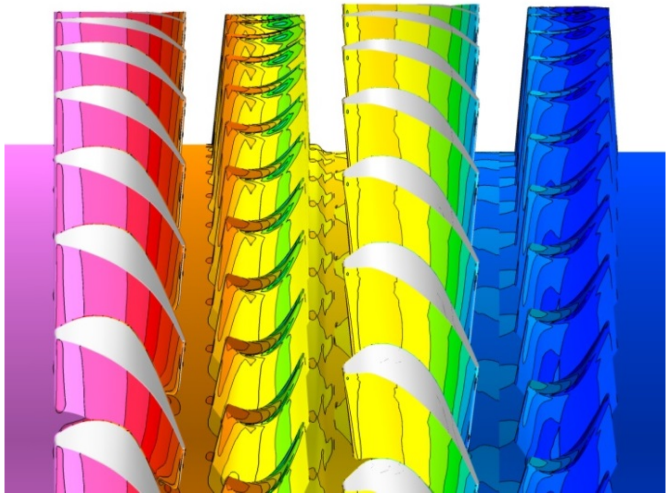

- **But main key points have to be solved to put CFD a step further**
  - Expertise of end-users still needed to provide « correct results »
  - Error margins still not mastered
    - Computational errors (Verification process)
    - Uncertainties : physical model & boundary/initial conditions
  - Efficiency (cost) in particular for unsteady CFD dealing with scale-resolving problems for new computers architecture
  - CFD used in a multiphysics environment

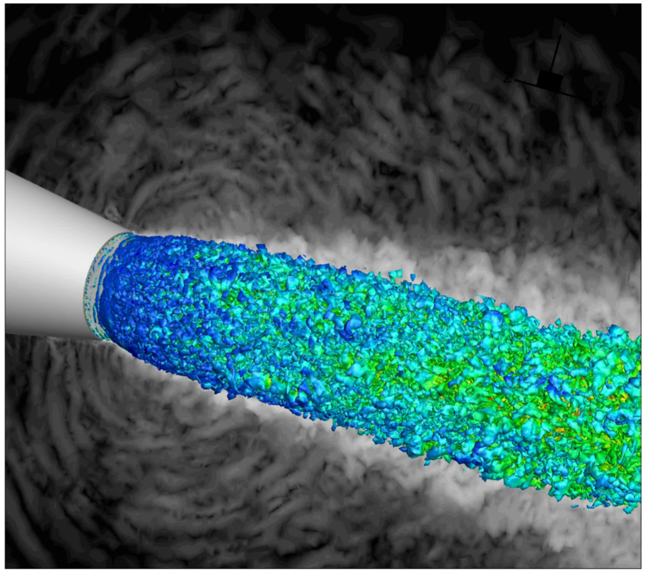


- **Automatization and Accuracy Control challenge :**
  **From End-users expertise to Expert CFD system**

# Processus de simulation numérique en mécanique des fluides







**Définition du cas de calcul**

**Modèle physique et données opérationnelles avec incertitudes**

- **Approche déterministe**
- **Approche stochastique**

→

**Code de calcul**

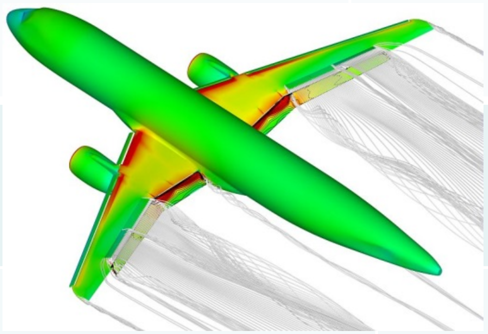
**Méthodes de calcul CFD avec approximation du modèle continue et complétude des conditions aux limites ou conditions initiales (par ex.)**

→

**Analyse**

**Résultats de simulation avec propagation des incertitudes et des sensibilités numériques**

# CFD – Enjeux scientifiques et techniques



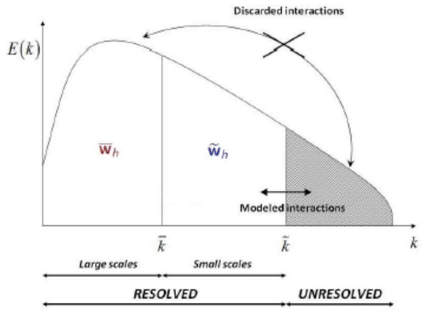
| | |
| --- | --- |
| S6 | **Maîtrise des systèmes complexes / Multiphysique précise** |
| S5 | **Optimisation robuste avec incertitudes** |
| S4 | **Optimisation de forme** ; **Quantification des incertitudes opérationnelles** ; **Contrôle des écoulements** |
| S3 | **Analyse de sensibilités**<br>- Paramètres du modèle<br>- Paramètres opérationnels ; **Multiphysique robuste** |
| S2 | **Calcul direct (analyse) avec vérification et validation**<br>- Solution analytique de (S1)<br>- Méthodes numériques standard pour approximation de (S1)<br>- Méthodes numériques de haute fidelité pour approximation de (S1) avec contrôle d'erreur |
| S1 | **Modèle physique / mathématique CFD**<br>(Euler, Navier-Stokes et sous-modèles associés) ; **Modèles structure, thermique, …** |

## Accroissement des capacités de simulation numérique

- Précision, efficacité et robustesse des algorithmes
- Optimisation méthodes de programmation / Architecture plateformes HPC
- Production de données massives (1 calcul DNS, des milliers de calcul RANS)

# Turbulence : Scales constraints

- ❑ Laminar flows / Turbulent flows
- ❑ DNS / LES / RANS




$$Reynolds\ number:\ Re_L = \frac{\rho V L}{\mu}$$

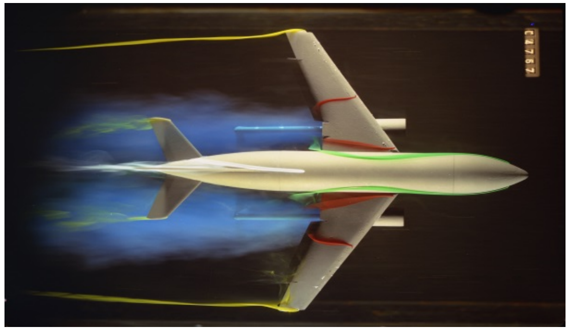
- ❑ DNS computations:
  - ❑ Kolmogorov scale = size of the smallest turbulent structure $\eta = L Re_L^{-3/4}$ per direction
  - ❑ With L integral scale representative of the larger turbulent structures
  - ❑ 1D computational domain with length N.h, N nomber of points, h mesh size, needs :

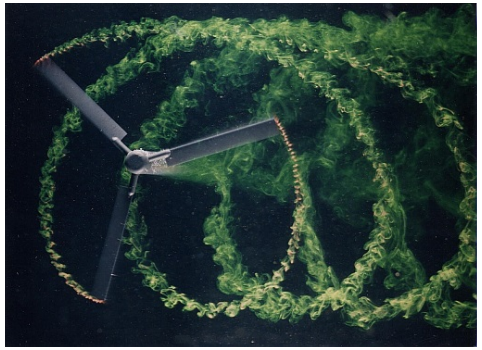
$$Nh \geq L$$

$$h \leq \eta$$

$$\eta N \geq NH \geq L$$

$$\Rightarrow N \geq L/\eta = \mathrm{Re}_L^{3/4}$$





- ❑ 3D DNS computation : $N_{3D} = Re_L^{9/4}$ , $Re_L = 10^6 \rightarrow N_{3D} \approx 3.10^{13}$
- ❑ Constraints on time step for integration of the Navier-Stokes systems (explicit schemes)
  - ▪ Δt proportional to $h$ for convective terms
  - ▪ Δt proportional to $h^2$ pour les termes de diffusion

# Simulation of turbulence : DNS or turbulence modelling

- Direct Numerical Simulation : Necessitate a very fine mesh to take into account all the scales of turbulence

- Introduction of models to take into accout phenomena with scales not represented in the mesh
  - Large Eddy Simulation : LES
  - Reynolds Averaged Navier-Stokes equations : RANS
  - Hybrid RANS/LES methods

- LES methods : Filtered Navier-Stokes equations

  $u = \tilde{u} + u''$ ,

  $\tilde{u}$ filtered (or resolved) field

  $u''$ modelled field (or modelled scales) : $\widetilde{u''} \neq 0$

- RANS : Averaged Navier-Stokes equations

  $u = \bar{u} + u'$ ,

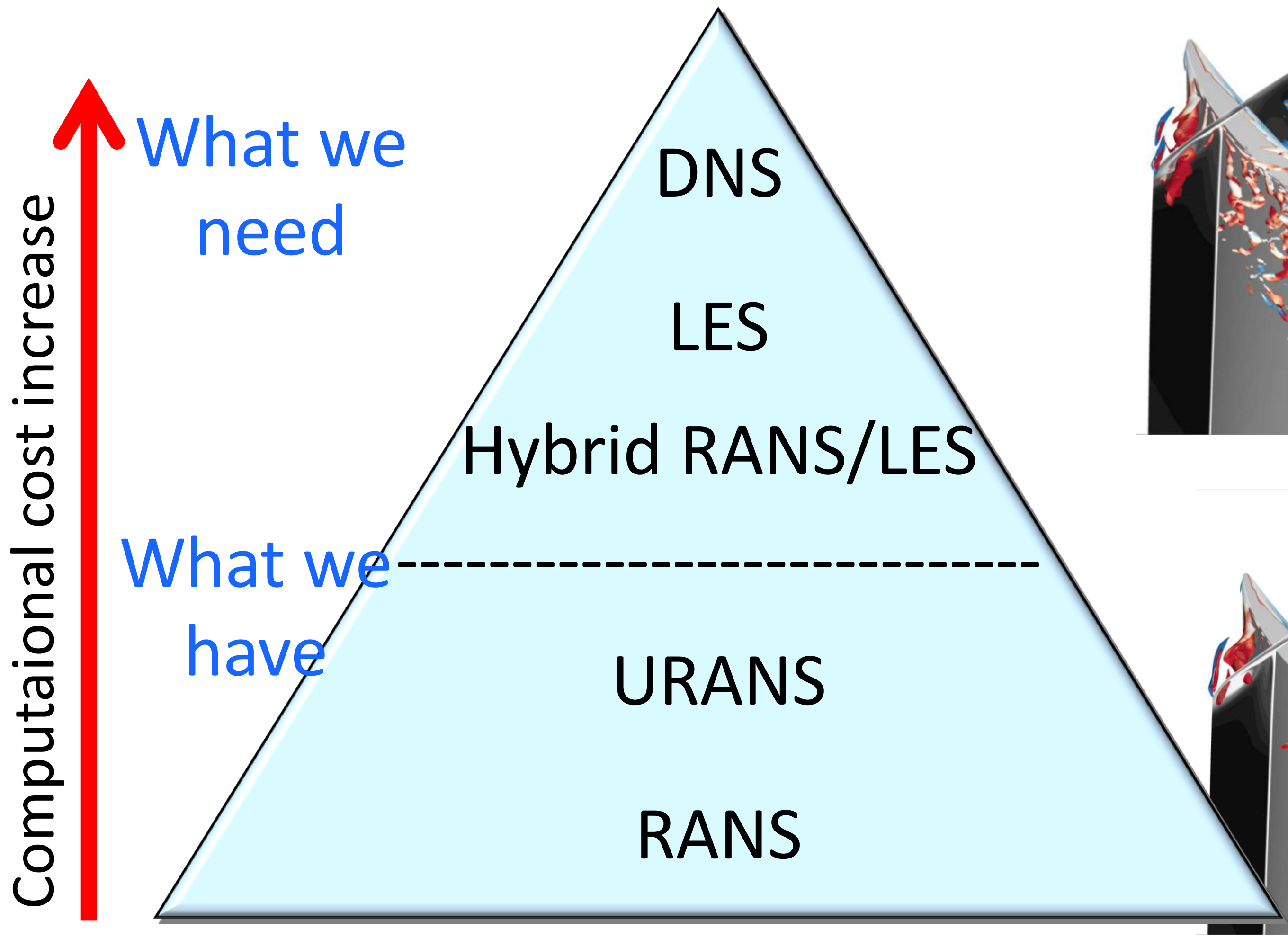
  $\bar{u}$ mean field

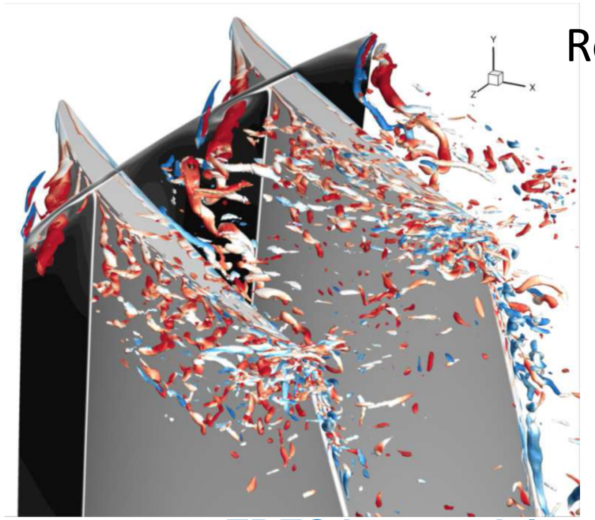
  $u'$ fluctuations : $\overline{u'} = 0$

  All the turbulent scales are modeled (non resolved)

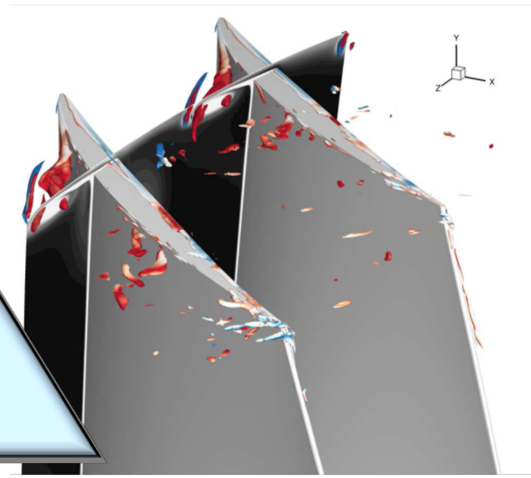
  $\bar{u}$ is the ensemble averaging (TBD)

# Turbulence modelling



ZDES k-ω model



Resolved

Modeled

SAS Menter Model

Comparison on tip-leakage vortex

# RANS : Reynolds Averaged Navier-Stokes system of equations

The RANS equations for compressible flows are (averaging notations removed)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{V}) = 0$$

$$\frac{\partial \rho \vec{V}}{\partial t} + \vec{\nabla} \cdot \left[ \rho \vec{V} \otimes \vec{V} + p\bar{\bar{I}} - \bar{\bar{\tau}} - \bar{\bar{\tau}}_R \right] = 0$$

$$\frac{\partial \rho E}{\partial t} + \vec{\nabla} \cdot \left[ \rho E \vec{V} + \left( p\bar{\bar{I}} - \bar{\bar{\tau}} - \bar{\bar{\tau}}_R \right) \cdot \vec{V} + \vec{q} + \vec{q}_R \right] = 0$$

It is necessary to model the following quantities in order to close the system

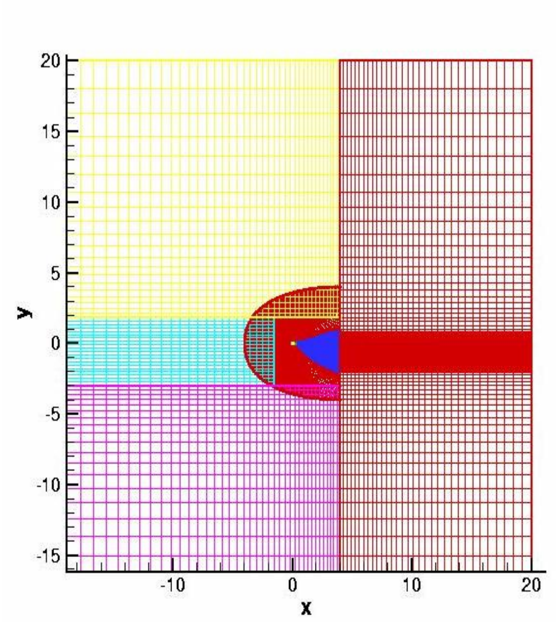
- Turbulent stress tensor (Reynolds tensor) : $\bar{\bar{\tau}}_R = \overline{-\rho \vec{V}' \otimes \vec{V}'}$
- Turbulent kinetic energy : $k = \frac{1}{2} \overline{\rho V'^2} / \bar{\rho}$ $\quad E = e + \frac{\vec{V}^2}{2} + k$
- Turbulent enthalpy flux : $\vec{q}_R = \frac{1}{2} \overline{\rho \vec{V}' h'}$

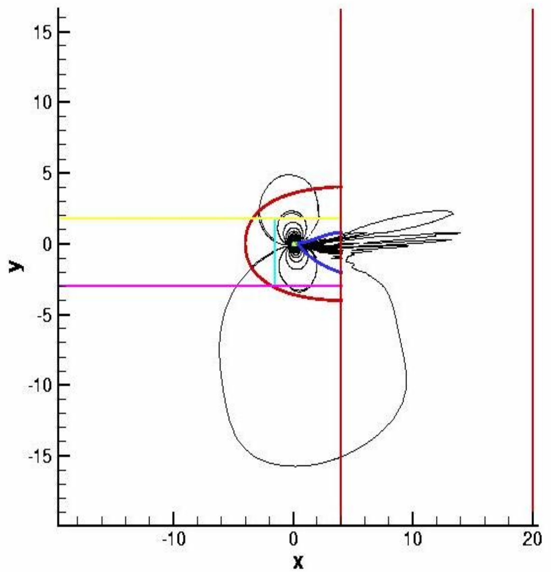
Turbulence model example : Wilcox K-ω model

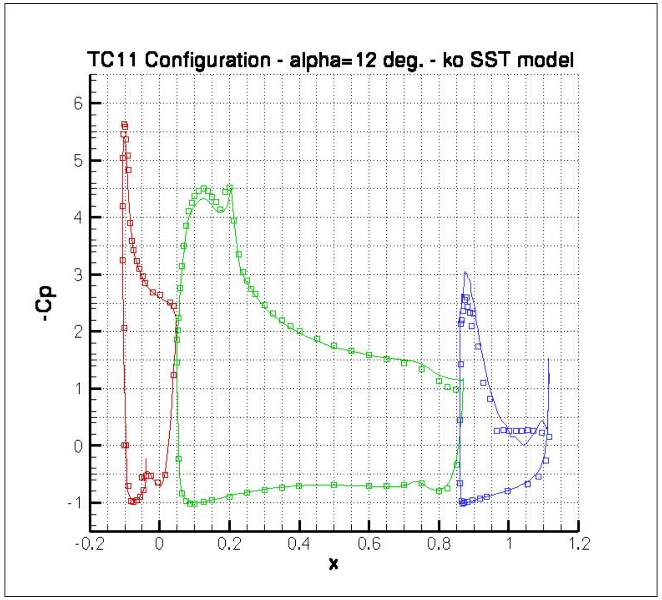
$$\frac{\partial \rho k}{\partial t} + \vec{\nabla}.(\rho \vec{V} k) = \vec{\nabla}.\left[ \left( \mu + \sigma^* \mu_t \right) \vec{\nabla} k \right] + \bar{\bar{\tau}}_t : \vec{\nabla} \vec{V} - \beta^* \rho k \omega$$

$$\frac{\partial \rho \omega}{\partial t} + \vec{\nabla}.(\rho \vec{V} \omega) = \vec{\nabla}.\left[ \left( \mu + \sigma \mu_t \right) \vec{\nabla} \omega \right] + \alpha \frac{\omega}{k} \bar{\bar{\tau}}_t : \vec{\nabla} \vec{V} - \beta \rho \omega^2$$

# Unsteady turbulent computations in non-matching meshes



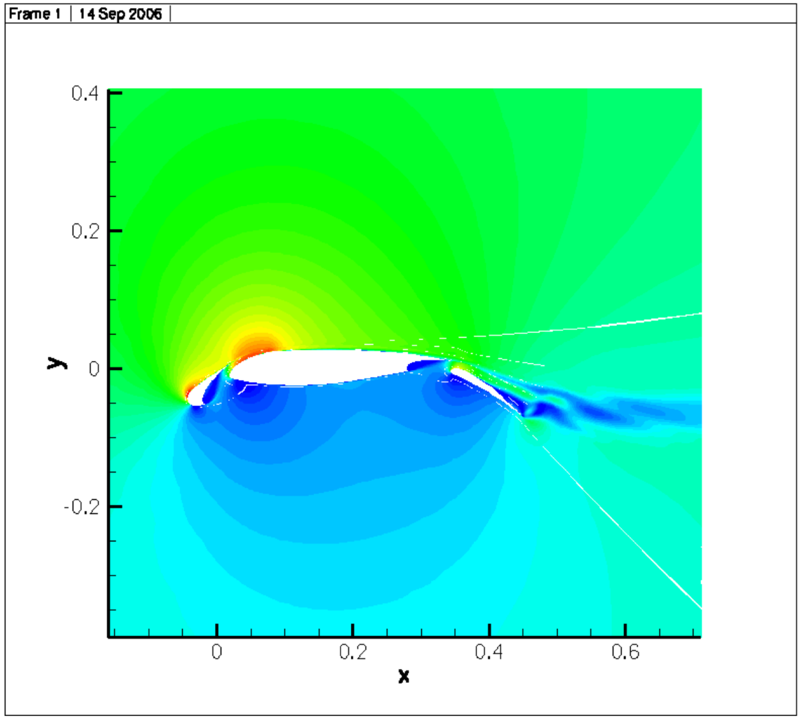



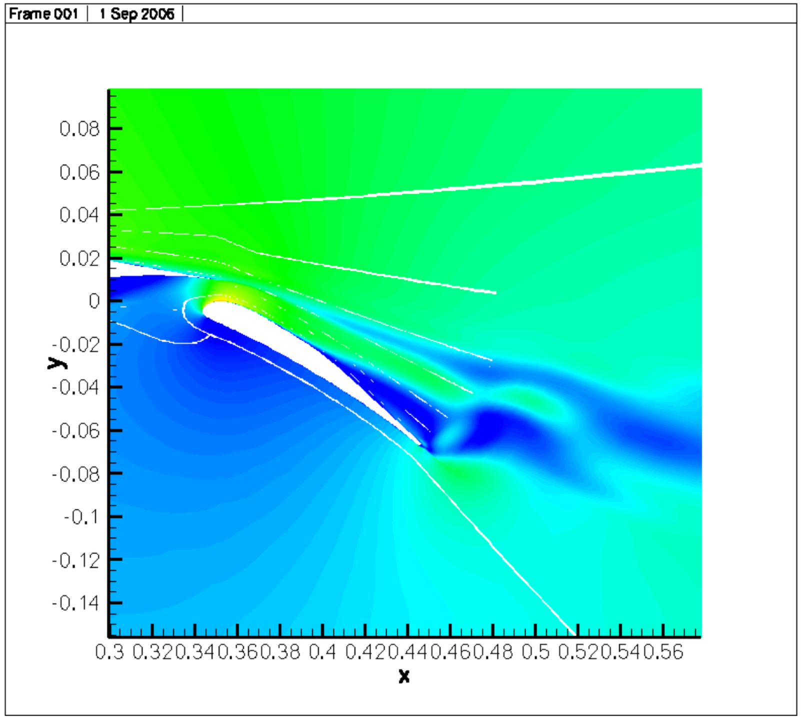




New extended mesh (50 chords) with refinement in the wake region to be consistent with the initial grid in the overlapping regions 50 chords

Cz higher with the extended mesh computation

# TC11 – URANS calculations – 12 Deg




SST Komega model – Unsteady computation
(the computation was first performed in a steady mode and did not converge; then it was performed in an unsteady mode using DTS)
K-Omega SST is in better agreement with the experiment than K-Omega Wilcox
Unsteady effects appear in the slat separation region

# Estimation d'erreur

- Estimation de l'erreur : Définition de la référence
  - Calcul / expérience : Barre d'erreur de mesure
  - Calcul / solution convergée (en DOFs) – Niveau de discrétisation espace/temps
  - Convergence itérative
  - Précision machine
  - Modélisation de la turbulence : LES (ref DNS filtrée ou DNS non filtrée) / filtrage explicite
  - Prise en compte des incertitudes : méthodes NIPCM : comment définir les pdf d'entrée ?
  - NIPCM et PCM : Approches non intrusives en modélisation
    - PCM : intrusive dans le code CFD
    - NIPCM : non intrusive
  - Synergie calcul / expérience ou référence
    - Fusion de données : non intrusif dans le code
    - Assimilation de données : intrusif dans le code pour prise en compte de données
    - IA pour modélisation de la turbulence

# Ordre élevé / Gain en précision

Mesure de l'erreur entre solution numérique et solution exacte : exemple sur une EDO d'ordre 1

$$\frac{du}{dx} = au \quad , \quad u(0) = 1$$

$$u_{exacte} = e^{ax} \quad , \quad u(0) = 1$$

$$D = [0,1] \quad , \quad h = \frac{1}{n}, n = (10, 20, \ldots, 7120)$$

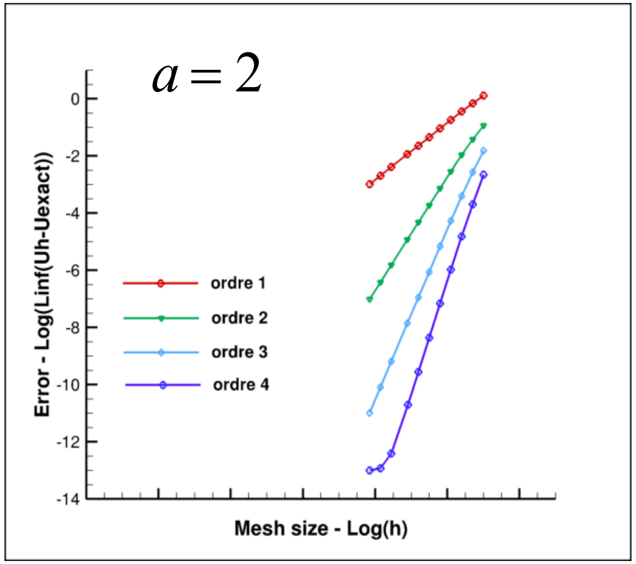
Branche asymptotique : $\underset{h \to 0}{u_{exacte} = u_h + Ch^p}$

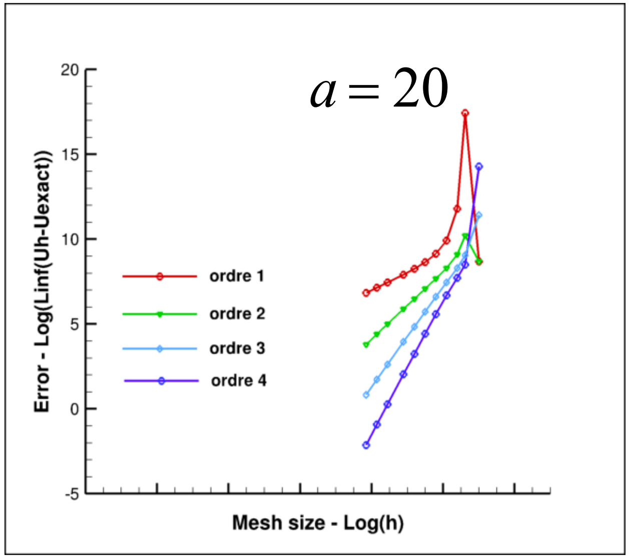
$$O1 : \quad u_{h,i} = \frac{u_{h,i-1}}{1 - ah}$$

$$O2 : \quad u_{h,i} = \frac{-u_{h,i-2} + 4u_{h,i-1}}{3 - 2ah}$$

$$O3 : \quad u_{h,i} = \frac{2u_{h,i-3} - 9u_{h,i-2} + 18u_{h,i-1}}{11 - 6ah}$$

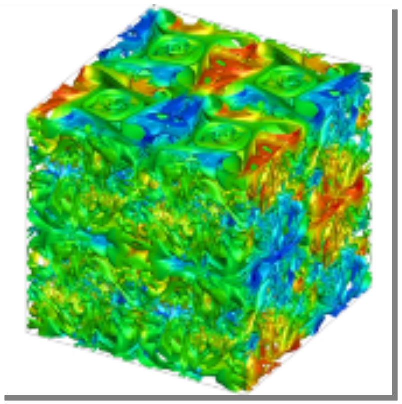
$$O4 : \quad u_{h,i} = \frac{-3u_{h,i-4} + 16u_{h,i-3} - 36u_{h,i-2} + 48u_{h,i-1}}{25 - 12ah}$$

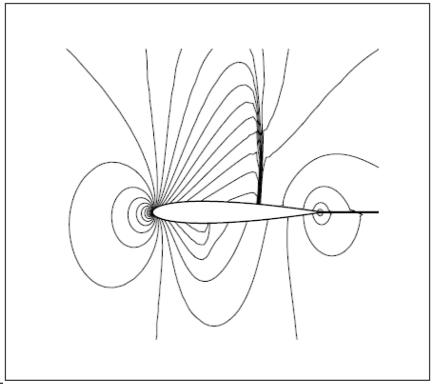


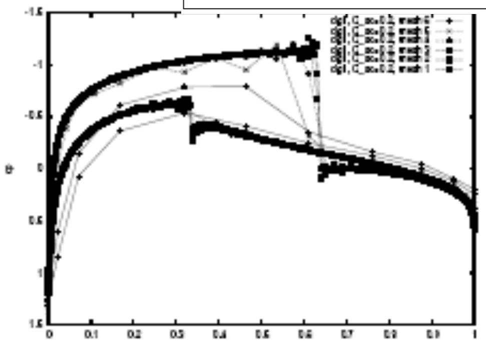



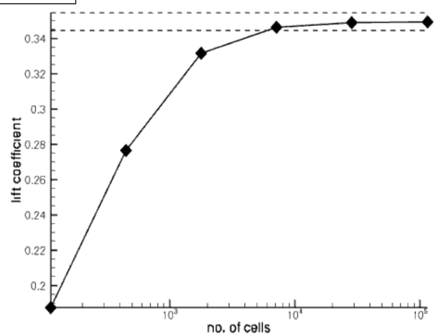
# Définition de la référence pour un calcul d'erreur

- Comparaison à des données expérimentales / comparaison à un solution idéale convergée (en raffinement espace/temps)
- Erreur liée au manque de résolution spatio-temporelle du calcul CFD
- Comparaison à des calculs/modèles de référence
- Marge d'erreur relatives aux données expérimentales
    - Conditions physiques de l'expérience : Incertitude / erreur de mesure
    - Complétude des données pour les conditions aux limites et les conditions initiales des calculs
    - …

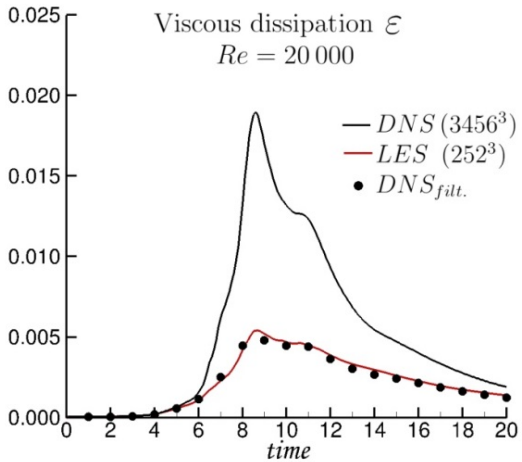




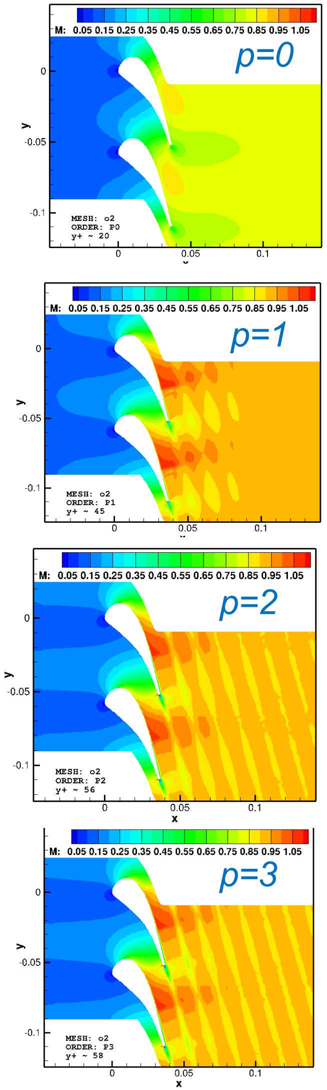





Convergence spatiale d'un calcul subsonique stationnaire

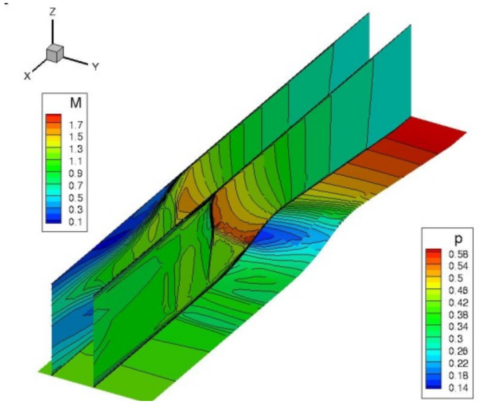


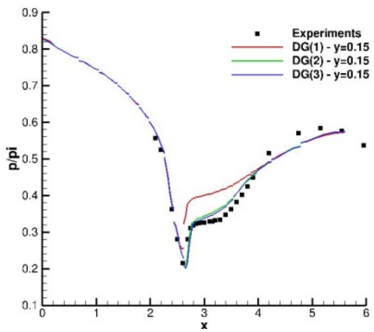

Comparaison LES à la référence DNS ou bien à la DNS filtrée ?

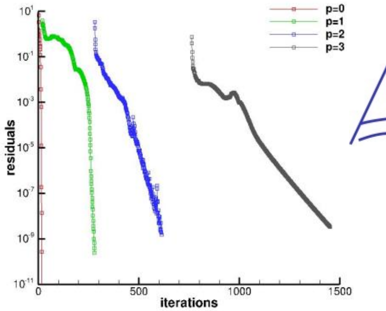



Transonic VKI turbine
at iso-Mesh : from p=0 to p=3

O. Labbé



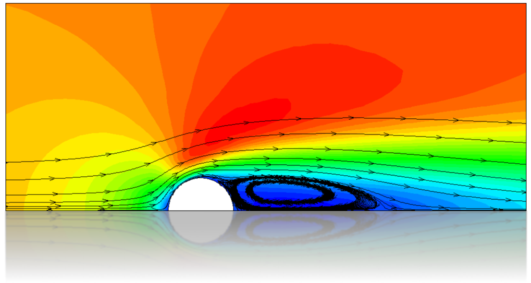



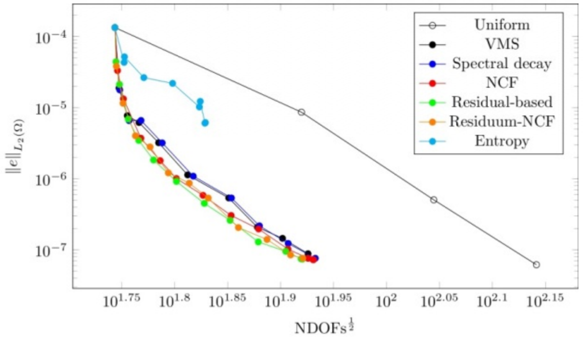



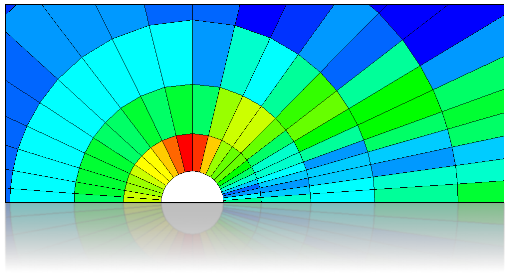

F. Renac

Aghora.

ONERA 3D swept Bump (Exp. J. Délery)
Efficient implicit method for HO-DG





**FD-09. CFD Solver Techniques**

AIAA-2018-0368.

*Fabio Naddei et al.*

Laminar cylinder - Efficiency of
local p-refinement with
various error indicators



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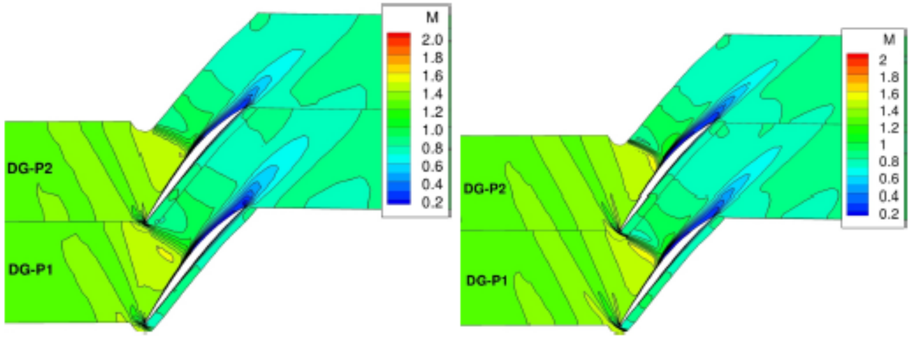
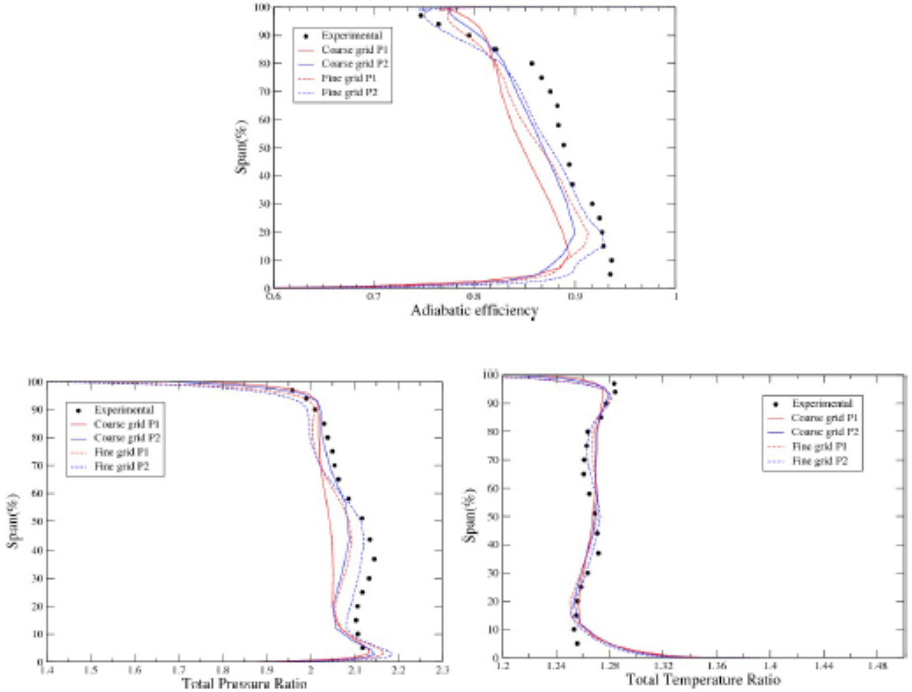
## DG-p1/p2 simulations on hexahedral meshes

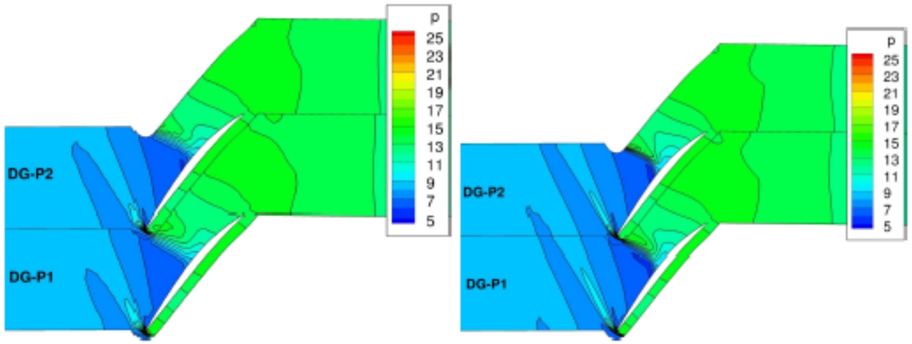
Coarse mesh with 87,769 points, fine mesh with 672,896 points.




Iso-contours of the Mach number (left: coarse, right: fine).




Spanwise profiles of adiabatic efficiency, total pressure ratio and total temperature ratio.

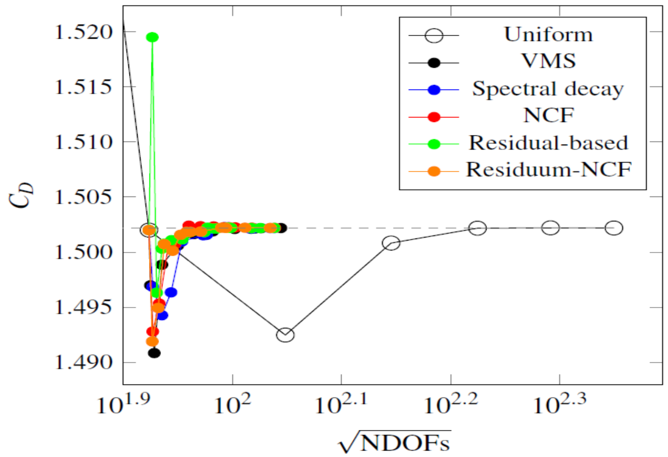



Iso-contours of the static pressure (left: coarse, right: fine).

*'A comparison of refinement indicators for p-adaptive simulations of steady and unsteady flows using discontinuous Galerkin methods'*, F. Naddei, M. de la Llave Plata, V. Couaillier, JCP 2019

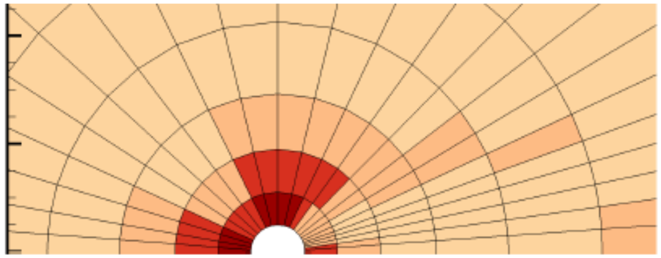
## Error estimators considered

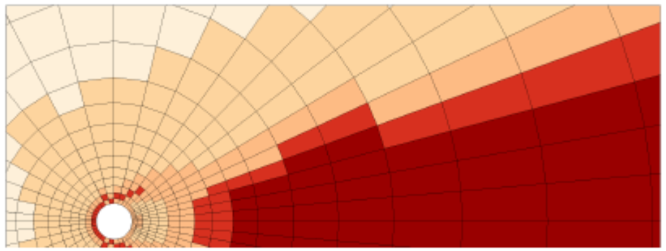
- Small-scale energy density (SSED) (Kuru *et al.* , 2016)
- Spectral decay (SD) (Tumolo *et al.* , 2013)
- Non-conformity (NCF) (Gassner *et al.* , 2009)
- Residual-based (RB) (Hartmann & Houston, 2002)
- Residuum-NCF (RNCF) (Dolejší *et al.* , 2013)



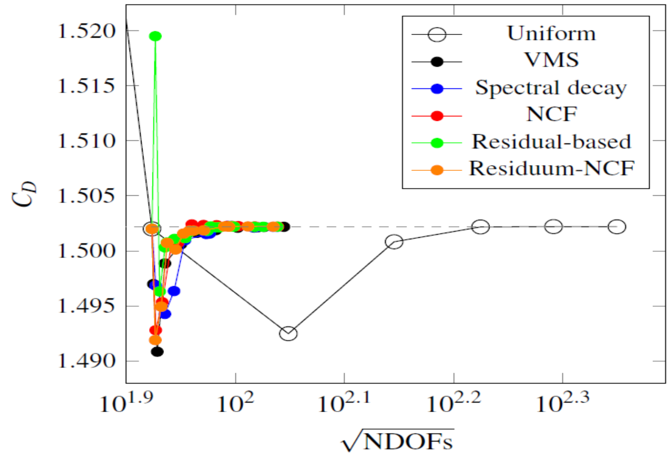

## Considered test cases

- **Euler steady**:
  Gaussian bump $M = 0.5$, Cylinder $M = 0.3$
- **Laminar steady**:
  Joukowski airfoil M= 0.5 Re= 1000,
  Cylinder M= 0.1 Re= 40
- **Laminar unsteady**:
  Cylinder M= 0.1 Re= 100





# Convergence analysis for steady laminar flow



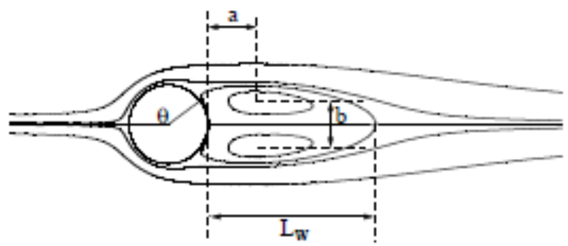

Aghora.

*Same Cd with margin error of* $10^{-4}$

*obtained with DG code Aghora, FV codes elsA and CANARI with h and/or p refinement and artificial external boundary extension*

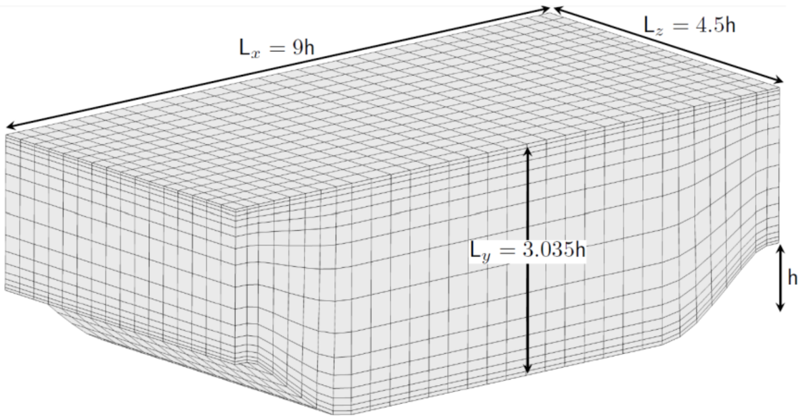
Table 1: Physical parameters of the flow pattern around a circular cylinder at $Re = 40$: Drag coefficient $C_D$, separation angle $\theta_s$, wake length $L_w/D$ and location of recirculation centre $(a, b)$.

| | $C_D$ | $\theta_s$ | $L_w/D$ | $a/D$ | $b/D$ |
|---|---|---|---|---|---|
| Tritton [1] | 1.48 | | | | |
| Dennis & Chang [2] | 1.52 | 126.2° | 2.35 | | |
| Coutanceau & Bouard [3] | | 126.2° | 2.13 | 0.76 | 0.59 |
| Fornberg [4] | 1.50 | 124.4° | 2.24 | | |
| He & Doolen [5] | 1.50 | 127.2° | 2.25 | | |
| Ye et al. [6] | 1.52 | | 2.27 | | |
| Calhoun [7] | 1.62 | 125.8° | 2.18 | | |
| Russel & Wang [8] | 1.60 | | 2.29 | | |
| Tseng & Ferziger [9] | 1.53 | | 2.21 | | |
| Linnick & Fasel [10] | 1.54 | 126.4° | 2.28 | 0.72 | 0.60 |
| Chung [11] | 1.54 | | 2.30 | | |
| Le et al. [12] | 1.56 | | 2.22 | | |
| Ding et al. [13] | 1.58 | 127.2° | 2.35 | | |
| Taira & Colonius [14] | 1.54 | 126.3° | 2.30 | 0.73 | 0.60 |
| Posdziec & R. Grundmann [15] | 1.49 | | | | |
| Patil & Lakshisha [16] | 1.56 | 127.3° | 2.14 | | |
| Bouchon et al. [17] | 1.50 | 126.6° | 2.26 | 0.71 | 0.60 |
| Present reference solution | 1.49 | 126.4° | 2.24 | 0.71 | 0.59 |




*R. Gautier, D. Biau, E. Lamballais : A reference solution of the flow around a circular cylinder at Rey = 40*
*https://hal.archives-ouvertes.fr/hal-00876327*

# P-adapted DNS of 2D hill flow at $Re_b$=2800




Geometric configuration (32x16x16 grid)

## Boundary conditions:

- Periodicity in the streamwise and spanwise directions
- Upper and bottom walls: isothermal $T_w = \frac{u_b^2}{\gamma R M_b^2}$

## Flow conditions:

- Quasi-incompressible regime: $M_b = 0.1$
- Reynolds number based on bulk velocity

$$Re_b = \frac{\rho u_b h}{\mu} = 2800$$

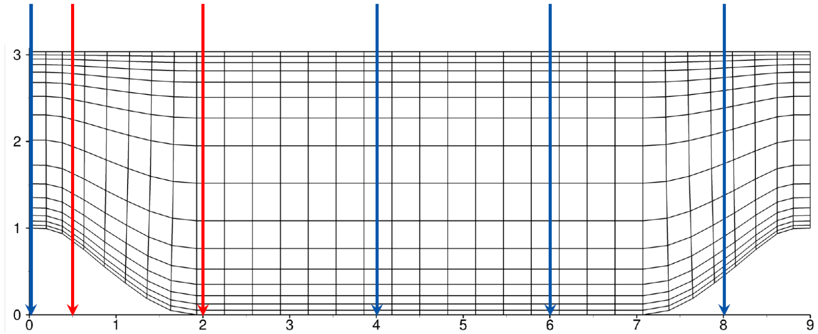
$$u_b = \frac{1}{2.035} \int_h^{3.035h} u(y)\mathrm{d}y$$

- Flow driven by pressure gradient:

$$\frac{\mathrm{d}p}{\mathrm{d}x}^{n+1} = \frac{\mathrm{d}p^n}{\mathrm{d}x} - \frac{1}{A_c \Delta t}(\dot{m}_0 - 2\dot{m}^n + \dot{m}^{n-1})$$

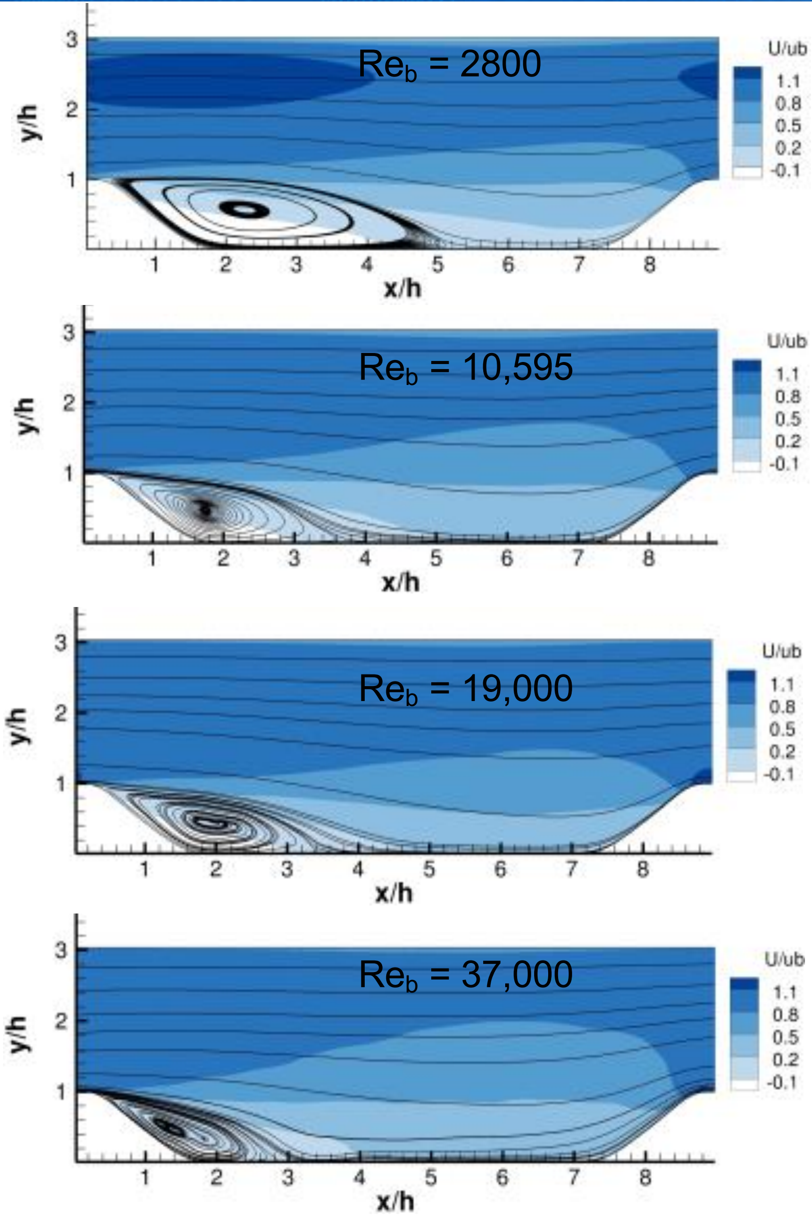
# P-adapted DNS of 2D hill flow at $Re_b$=2800

- Solution averaged over a minimum of $65t_c$ (convective time)
- Mean velocity and Reynolds stresses extracted at 6 locations



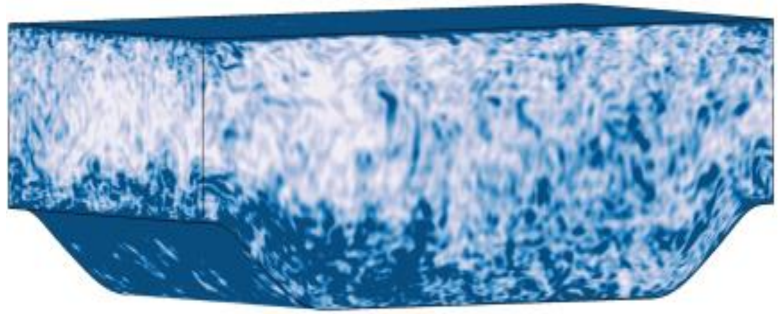

- Reference data: DNS, incompressible $2^{nd}$ order FV, $13.1 \cdot 10^6$ *DOFs* (Breuer *et al.* , 2009)
- This configuration has been extensively studied with uniform $h$ and $p$ refinement with *Aghora* (de la Llave Plata *et al.* , 2017)
- Refinement algorithm starts from uniform $p = 3$ ($0.52 \cdot 10^6$ *DOFs*)
- Highest resolution of adapted simulations $\sim 1.3 \cdot 10^6$ *DOFs*

# Periodic hill – instantaneous field and mean flow




## Ecoulements instantanés

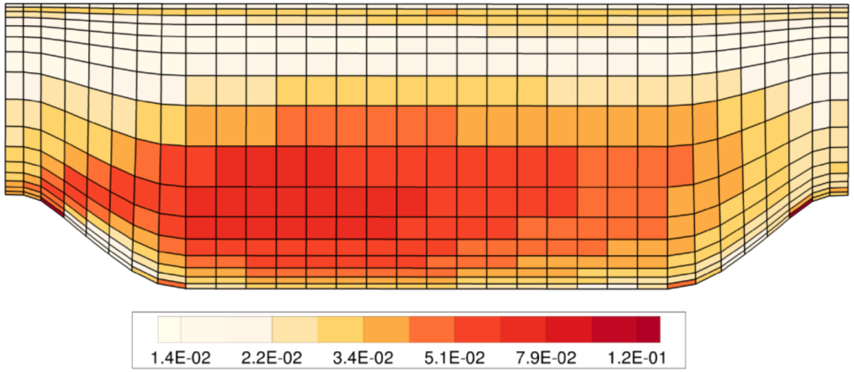
$Re_b$ = 2800



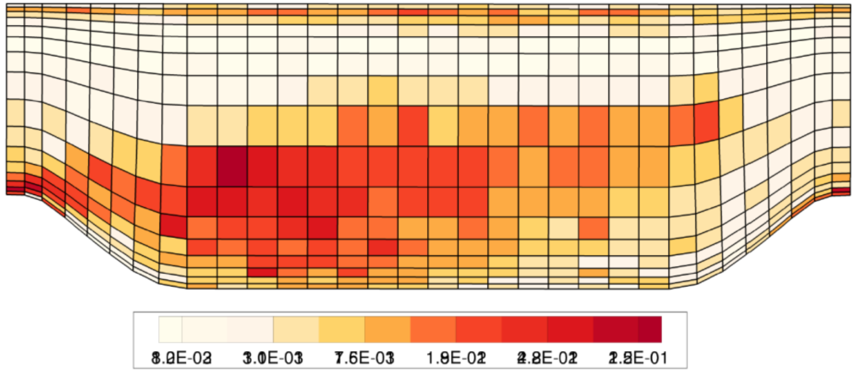
$Re_b$ = 10,595

*M. De la Lllave Plata*

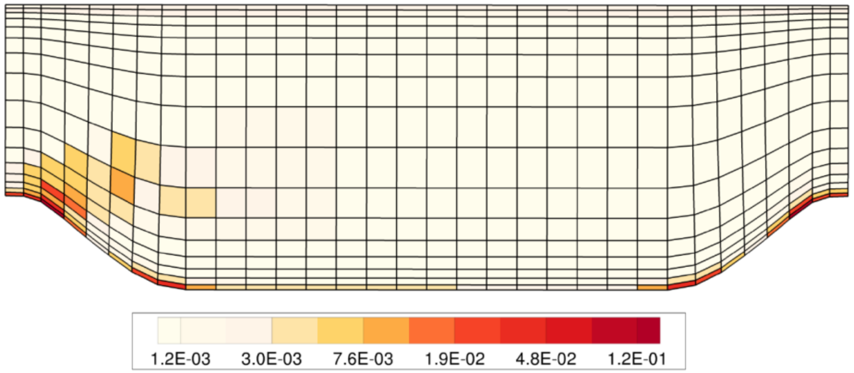
# Initial error distribution (p=3)




$L^2$-norm of error estimate



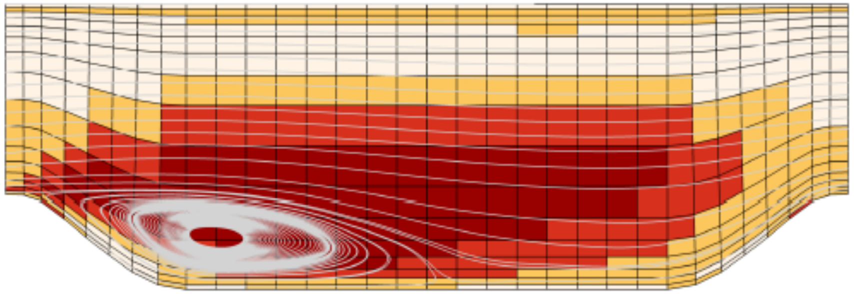
Maximum of error estimate



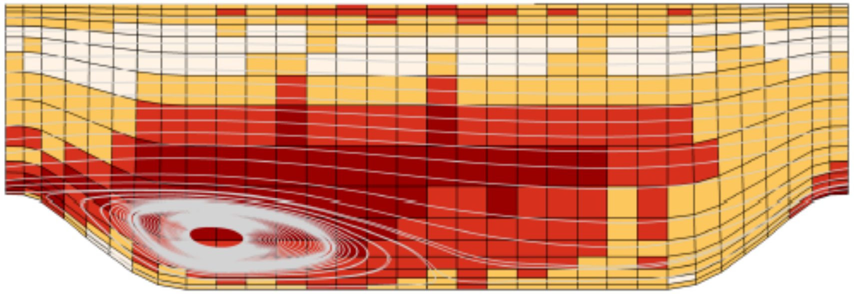

Error estimate from mean solution

*F. Naddei*

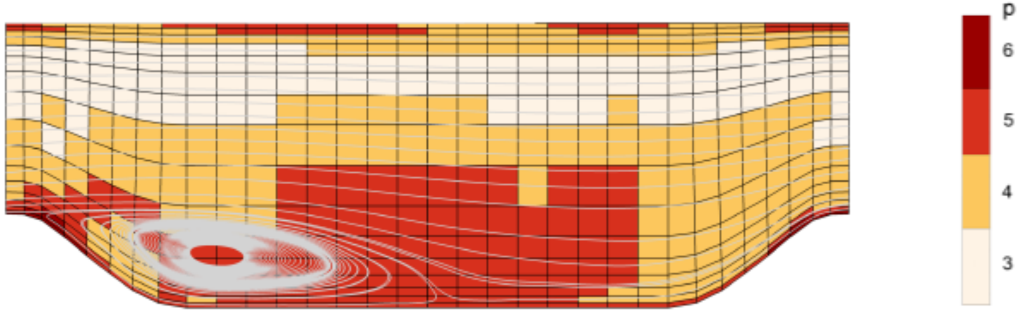
# Evolution of refinement levels : P3-P6



$L^2$-norm of error estimate



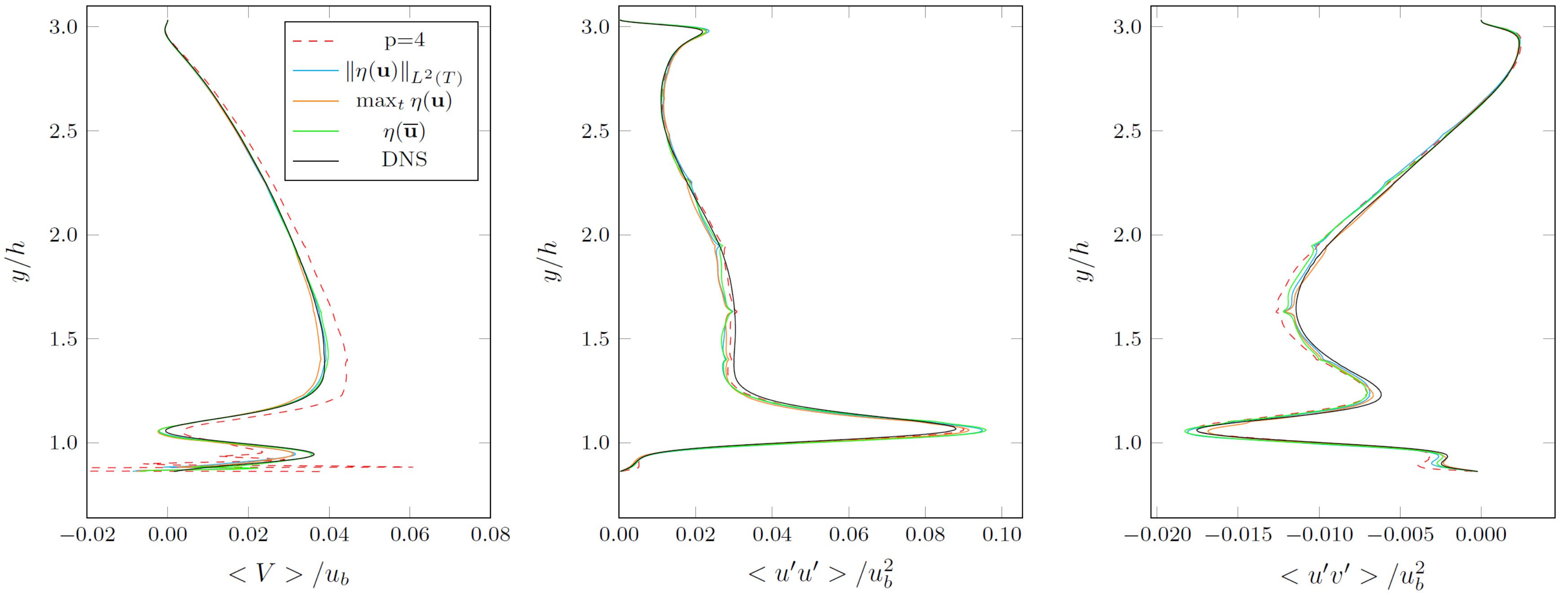
Maximum of error estimate



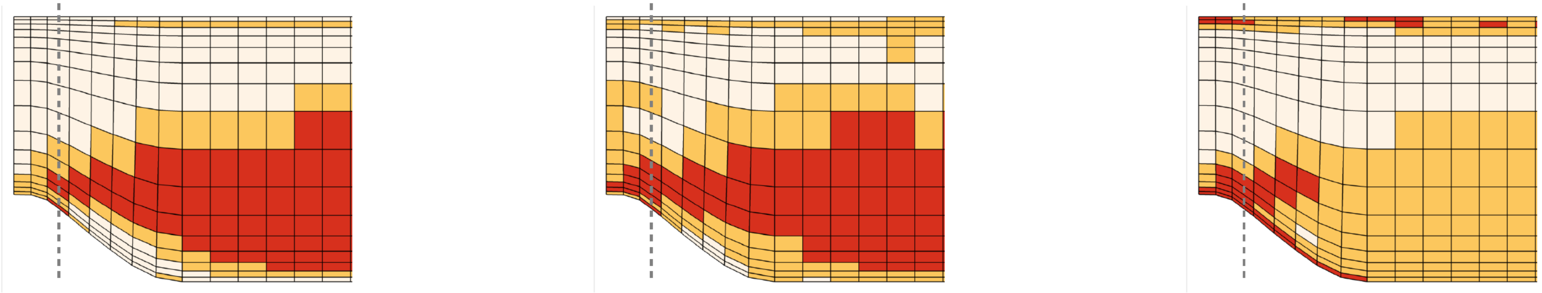

Error estimate from mean solution

| | $\eta_K^{L2}$ | $\eta_K^{M}$ | $\eta_K^{A}$ |
|---|---|---|---|
| $\#DOFs$ | $1.19M$ | $1.32M$ | $1.22M$ |

# Adaptive results at x=0.5 (near separation)




| | p=4 | $\eta_K^{L2}$ | $\eta_K^{M}$ | $\eta_K^{A}$ |
|---|---|---|---|---|
| $\#DOFs$ | $1.02M$ | $0.87M$ | $0.98M$ | $0.92M$ |



$\|\eta(\mathbf{u})\|_{L^2(T)}$ $\max_t \eta(\mathbf{u})$ $\eta(\overline{\mathbf{u}})$

# Propagation of uncertainties : Polynomial Chaos Methods

$\xi$ Input Random variable : Geometry, boundary conditions, initial conditions for unsteady problems
Scalar parameter in a RANS turbulence model

$\xi \rightarrow D(\xi)$ Probability Density Function in a set $\Gamma$ known or defined a priori

Evaluation of the moments of an output fonction *U* defined in F

- Could be a space-time function as a velocity component of an unsteady CFD computation
- Practically in CFD *U* is often a scalar integral quantity : Drag or lift coefficient for aircraft flows, Pressure ratio for turbo-engines, …
- The deterministic CFD computations are performed only for a finite number of $\xi$ values in $\Gamma$ leading to a discrete representation of ($\xi$ ,U) in the space $\Gamma \otimes$F
- Need to use a surrogate model to reconstruct U on the full space

$$u(x,t,\xi) = \sum_{i=0}^{p} u^i(x,t)\psi_i(\xi) \qquad (1)$$

$\psi_i$ A set of polynomial consistent with the pdf D in order to get an orthogonal basis

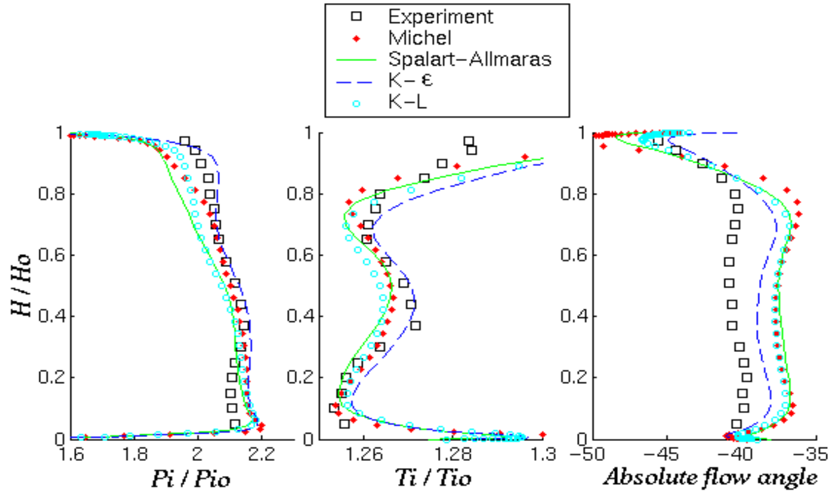
- For intrusive Polynomial Chaos Methods the unknowns of the PDE in the CFD code are replaced by their expansion defined in (1)

# NASA Rotor 37 : Propagation of uncertainties

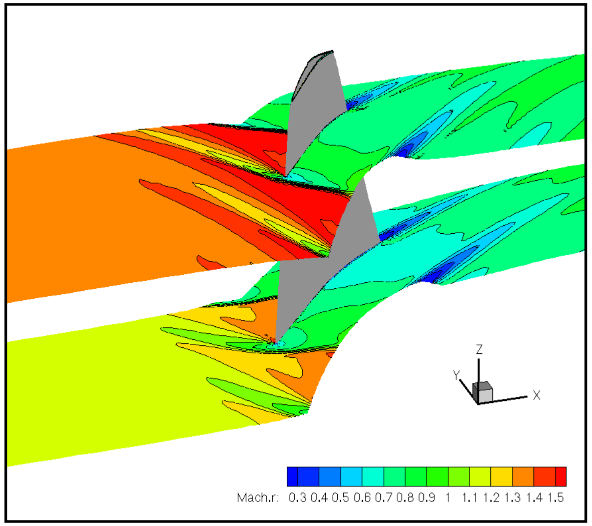
- NIPCM : Propagation of uncertainties for the computation of the pressure ratio
- ξ1 : outlet pressure
- ξ2 : Tip gap

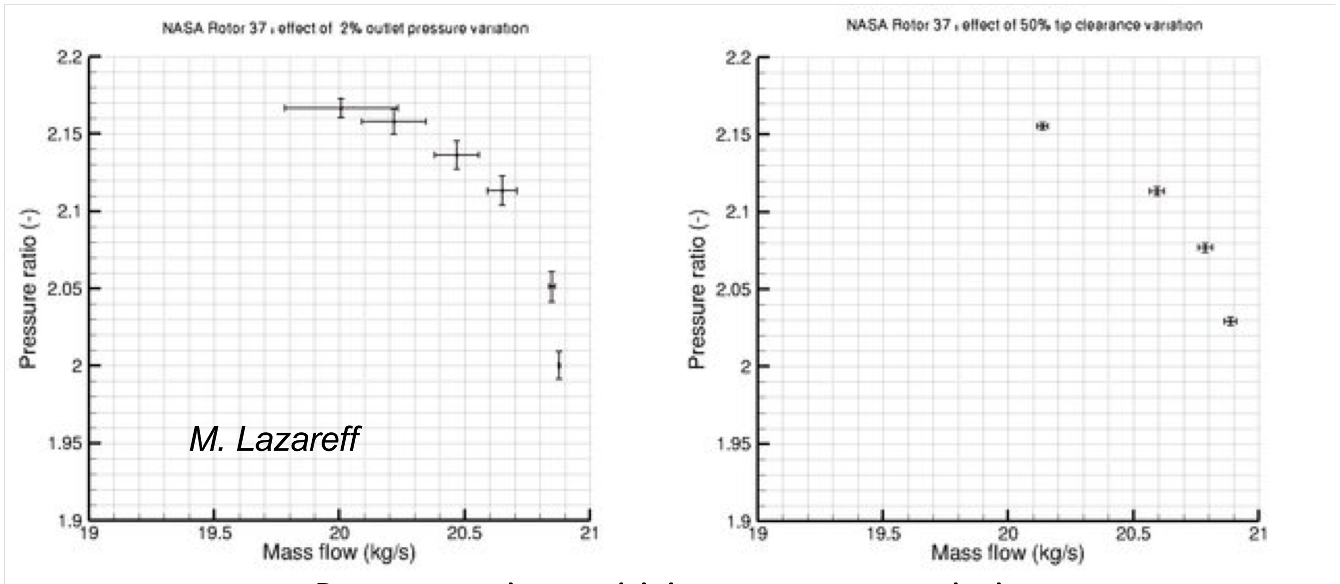
$$g(\xi) = \sum_{i=0}^{p} g^i \psi_i(\xi) \qquad (2)$$

PDF : Symmetric beta distribution
Jacobi polynomials with compact support



Deterministic computations with different turbulence models

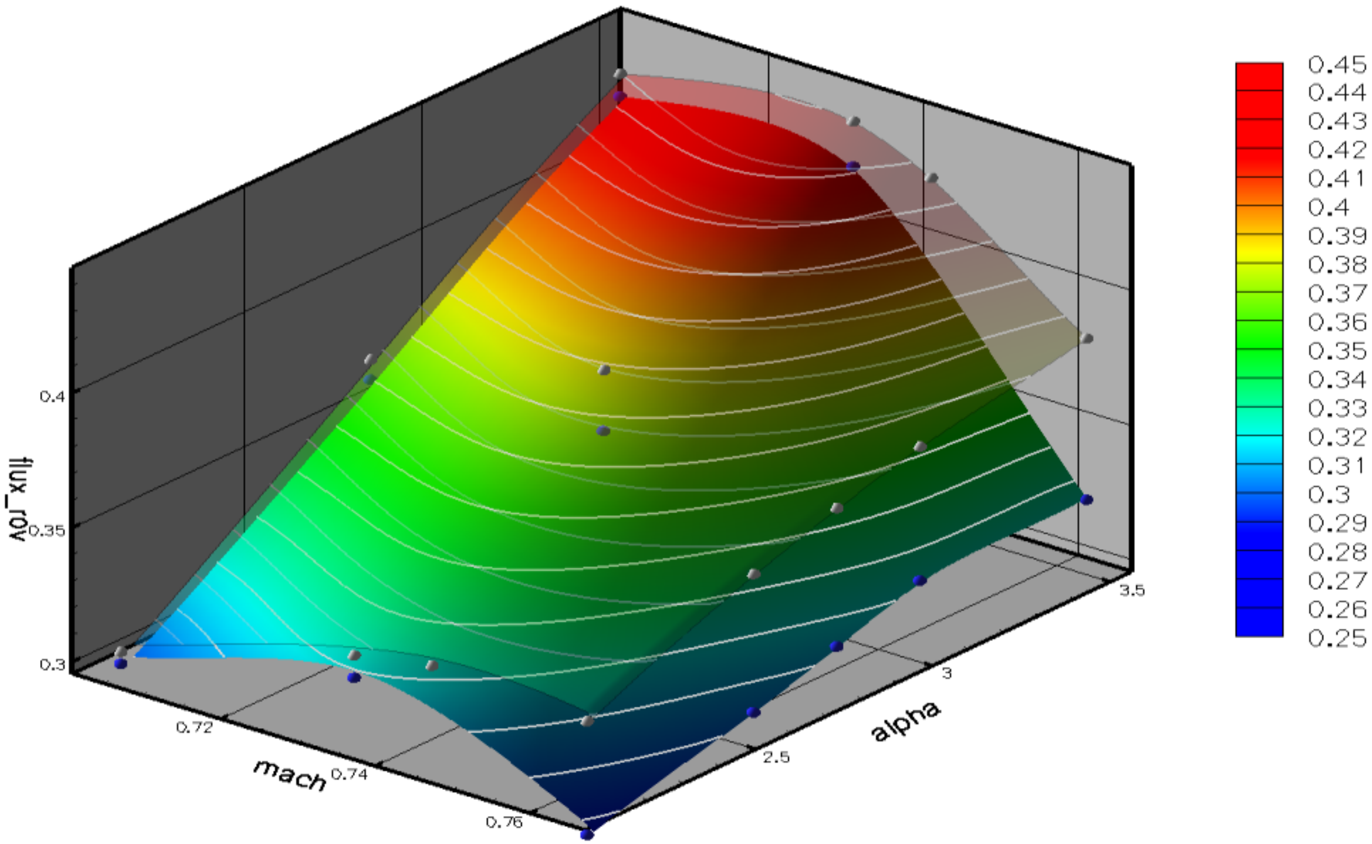


Pressure ratio sensitivity versus uncertainties

# RAE2822 Airfoil : Surrogate models for 2 turbulence models

Airfoil RAE2822 : Surrogate models using Kω or Kω-SST turbulence model



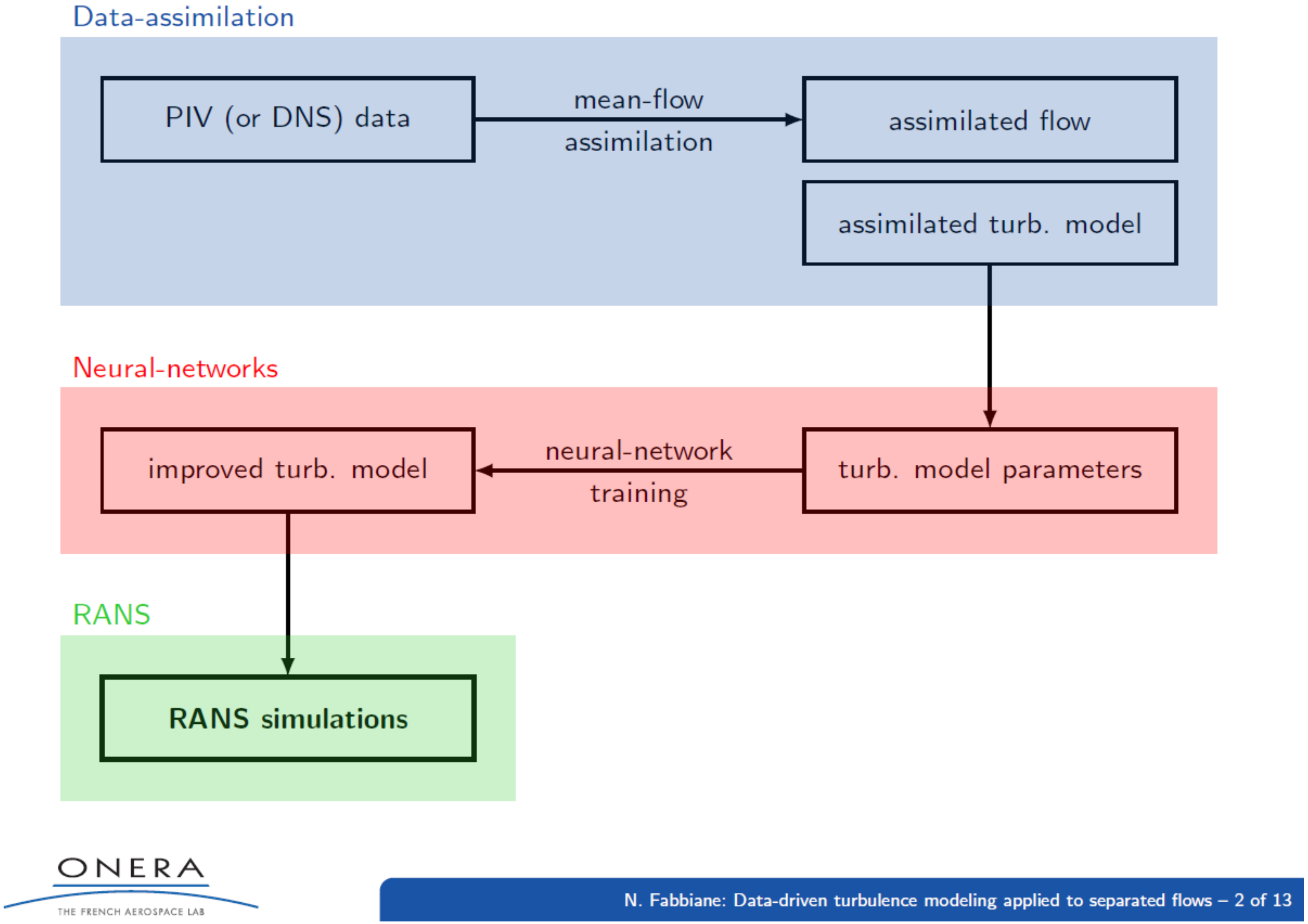

(Marc Lazareff)

# Turbulence model & Maching learning

- Improvement of RANS turbulence modeling to get as much as possible the quality of DNS/LES modelling
  - On mean flow quantities, integral/global quantities
- Use of reference data
  - Well-resolved DNS/LES computations of « simple flow configurations »
  - Experimental data : at ONERA project on synergy CFD/Experiments
  - Flight data
  - …
- Based on Data Assimilation methods
- Based on Machine learning, …
- Apply Improved turbulence models on advanced / complex flow configurations

**Data-driven turbulence modeling applied to separated flows**

Lucas Franceschini, **Nicolo Fabbiane**, Olivier Marquet, Benjamin Leclaire, Julien Dandois and Denis Sipp



# Data-driven turbulence modeling applied to separated flows

## A correction of Spalart-Allmaras model

Incompressible Reynolds-Averaged Navier-Stokes (RANS) with Spalart-Allmaras (SA) model:

$$\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \nabla\cdot\left((\nu+\nu_t(\tilde{\nu}))\left(\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T\right)\right)$$

$$\nabla\cdot\boldsymbol{u} = 0$$

$$\boldsymbol{u}\cdot\nabla\tilde{\nu} = \beta(\boldsymbol{x})\underbrace{c_{b1}\,\tilde{S}(\boldsymbol{u},\tilde{\nu})\,\tilde{\nu}}_{P(\boldsymbol{u},\tilde{\nu})\text{ production}} + \underbrace{\frac{1}{\sigma}\nabla\cdot((\nu+\tilde{\nu})\nabla\tilde{\nu})}_{D(\tilde{\nu})\text{ dissipation}} - \underbrace{c_{w1}\,f_w(\boldsymbol{u},\tilde{\nu})\left(\frac{\tilde{\nu}}{d}\right)^2}_{W(\boldsymbol{u},\tilde{\nu})\text{ destruction}}$$

Duraisamy and Singh (2016): use $\beta(\boldsymbol{x})$ to correct the production term $P(\boldsymbol{u},\tilde{\nu})$.

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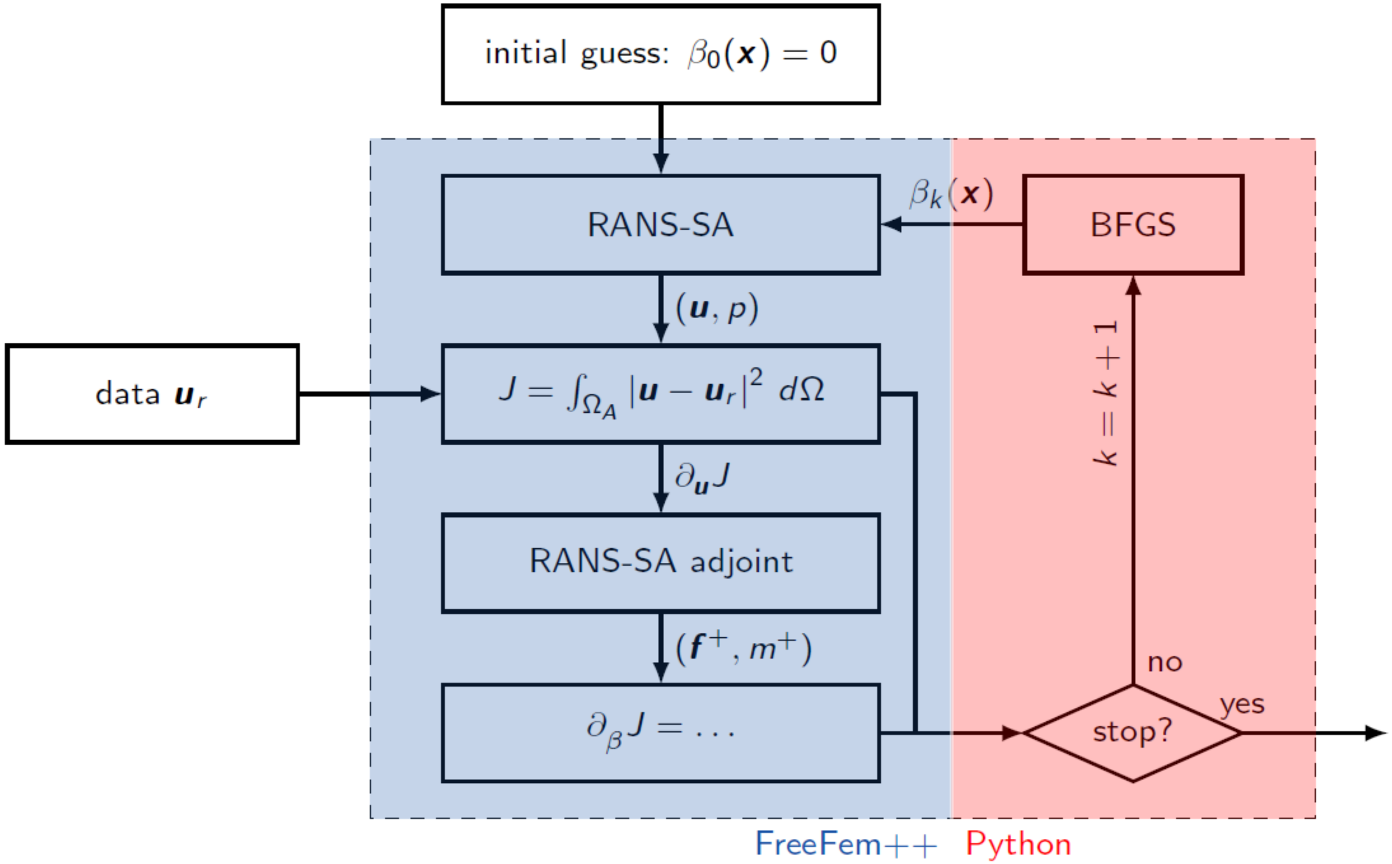
## Adjoint-based optim. . . assimilation

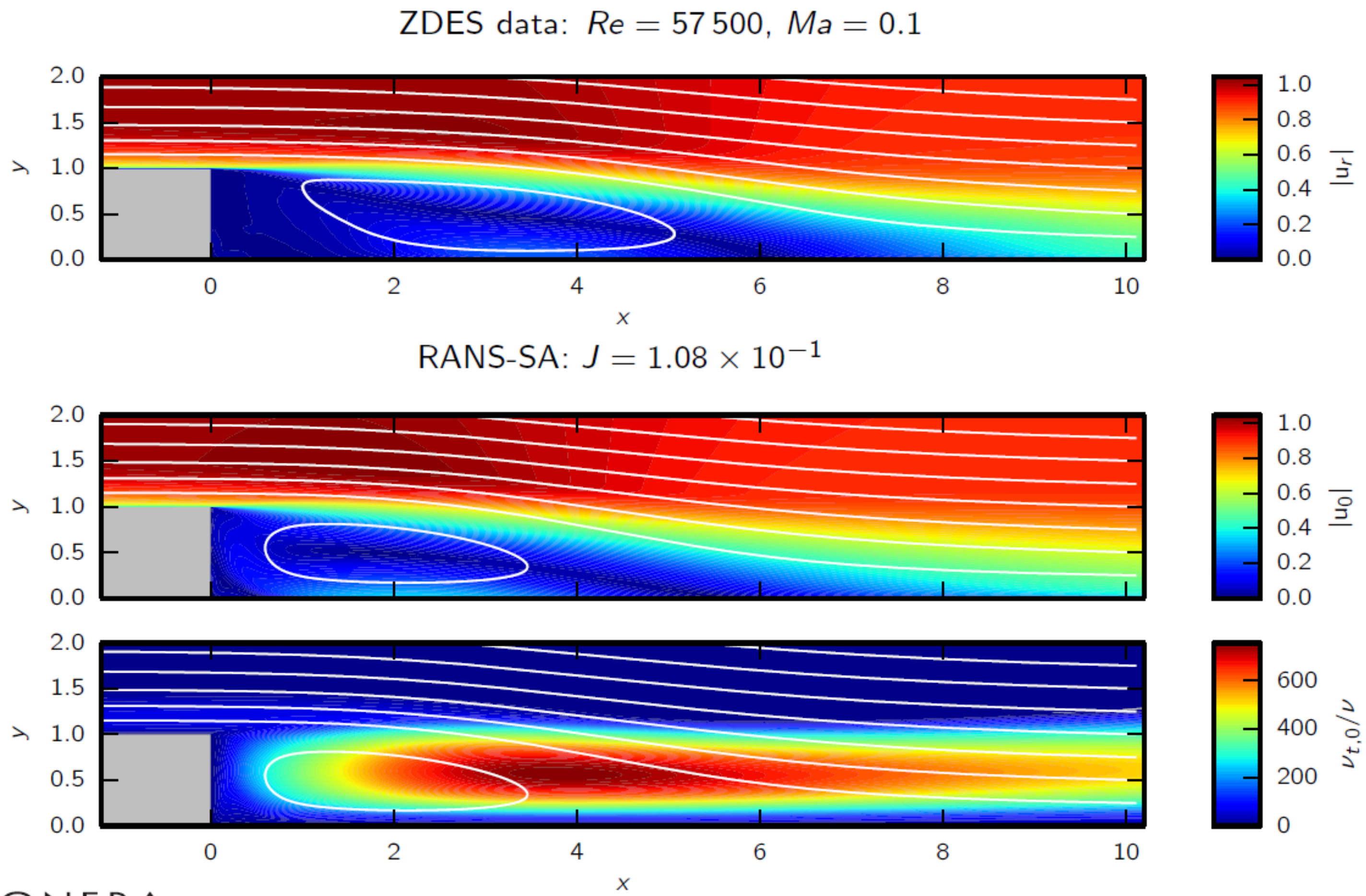
Adjust the production mask $\beta(\boldsymbol{x})$ to match the data $\boldsymbol{u}_r$, i.e. minimize the cost function $J$.



# Data-driven turbulence modeling applied to separated flows

## Flow assimilation (I)

**Data:** back-facing step simulations by Beneddine et al. (2016)

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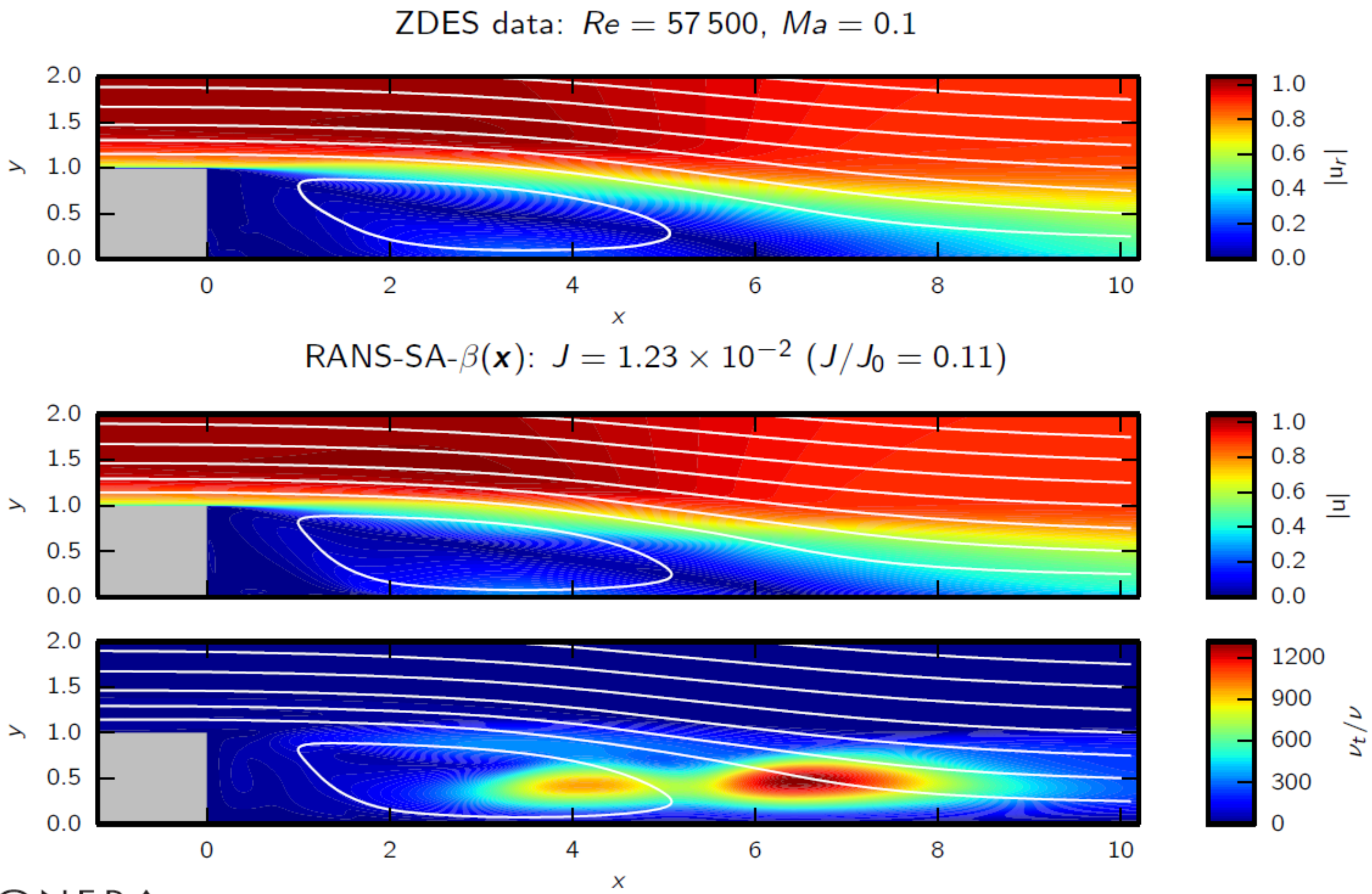
N. Fabbiane: Data-driven turbulence modeling applied to separated flows – 6 of 13

## Flow assimilation (I)

**Data:** back-facing step simulations by Beneddine et al. (2016)

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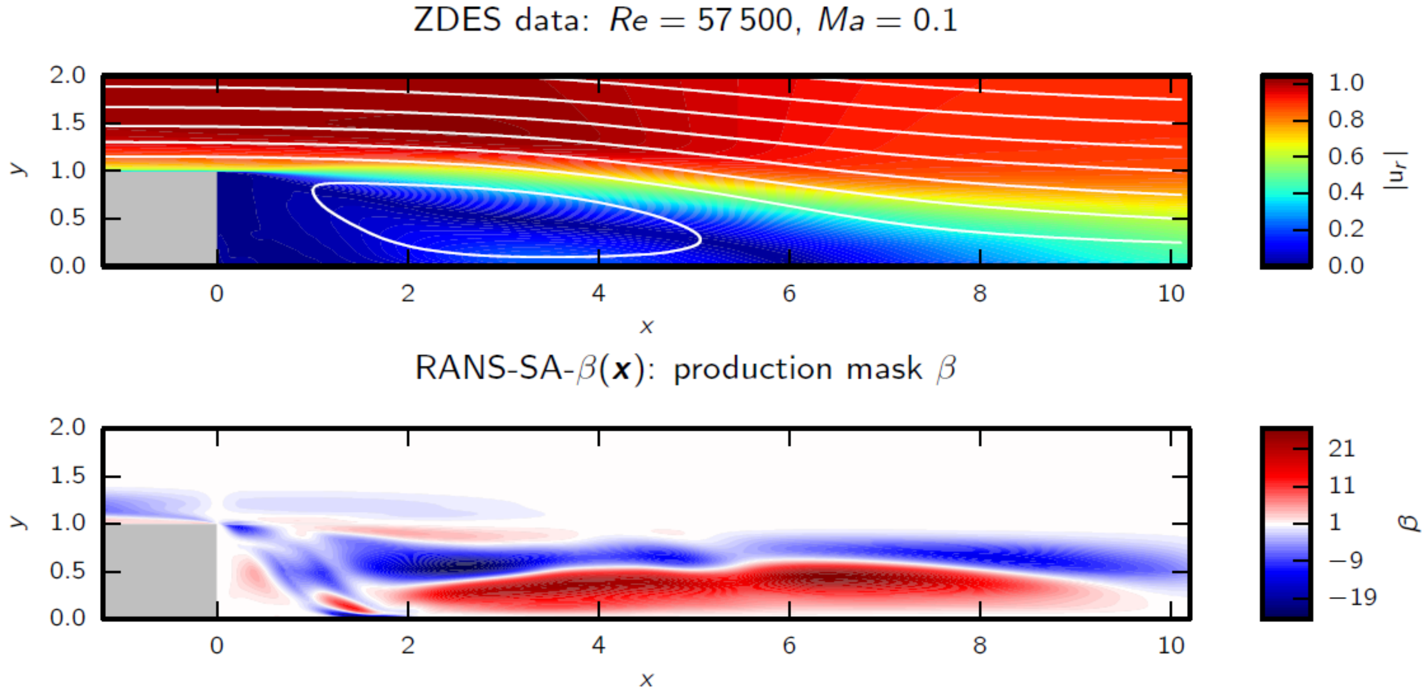
## Flow assimilation (I)

**Data:** back-facing step simulations by Beneddine et al. (2016)




$|\beta| \approx 20$ ▶ not a *small* correction of Spalart-Allmaras anymore!

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## Neural network

Produce a general input/output (I/O) function that reproduces the SA correction.

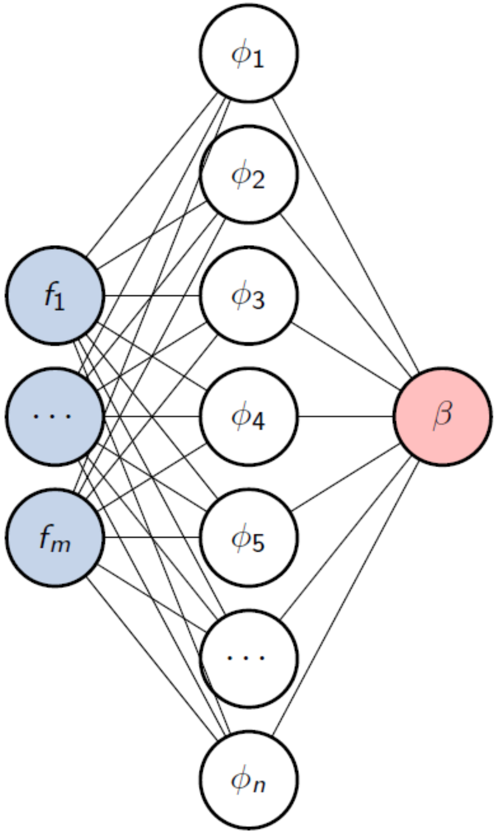
$$\boxed{\beta(\boldsymbol{x}) \rightarrow \beta(\boldsymbol{u}, \tilde{\nu}) \rightarrow \beta(\{f_j(\boldsymbol{u}, \tilde{\nu})\})}$$

- Not directly function of the state $(\boldsymbol{u}, \tilde{\nu})$ but on a set of observables $\{f_j(\boldsymbol{u}, \tilde{\nu})\}$
- Neural network (`scikit-learn` package): 1 layer of n=100 neurons.

$$\phi_i = \max\left(0, b_i + \sum_{j=1}^{m} a_{ij}\, f_j(\boldsymbol{u}, \tilde{\nu})\right)$$

$$\beta = \beta_0 + \sum_{i=1}^{n} w_i \phi_i$$

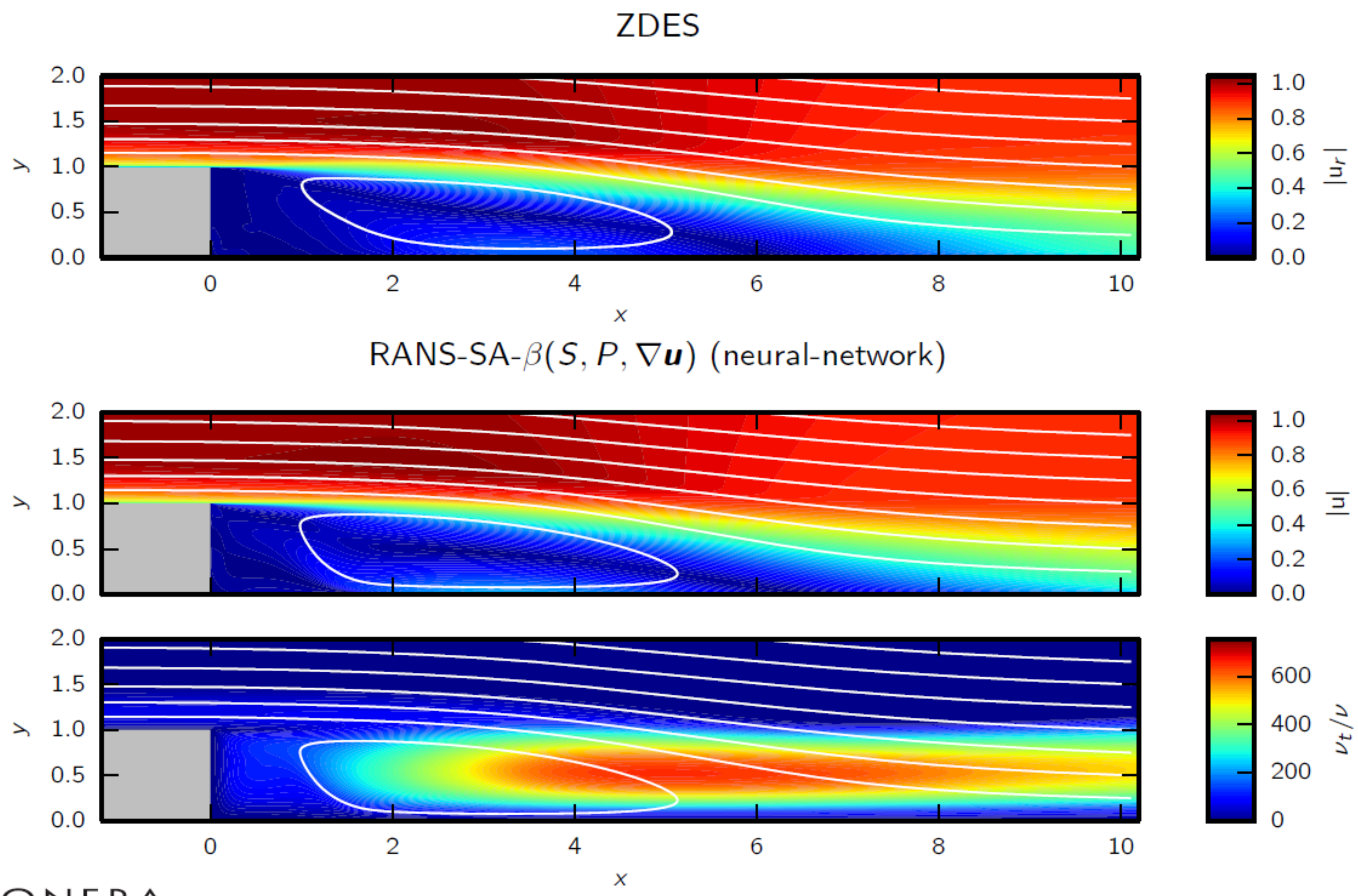
- Which observables $f_j(\boldsymbol{u}, \tilde{\nu})$?
  First attempt: $\boxed{\{f_j\} = \{\ln(S), P, \nabla \boldsymbol{u}\}}$



# Data-driven turbulence modeling applied to separated flows

- Data assimilation : $\beta(x)$ for a given geometrical and physical configuration
- Machine learning using Neural network : Define $\beta^*(u,\tilde{\vartheta})$ to be used for various flow configurations
- $\beta^*(u,\tilde{\vartheta})$ define using $\{f_j(u,\tilde{\vartheta})\} = \{\ln(S), P, \nabla u\}$ at the training points
- Minimize a norm of $[\beta^*(\{f_j(u,\tilde{\vartheta})\}) - \beta(x)]$ at the training points using back propagation
- Variables in blue (previous slide) are the quantities to quantities defined by the training process
- Activation fonction : Max (0, entry)
- Back-facing step :
    - 30,000 points for RANS computation
    - 7,500 for training with NN

## Validation

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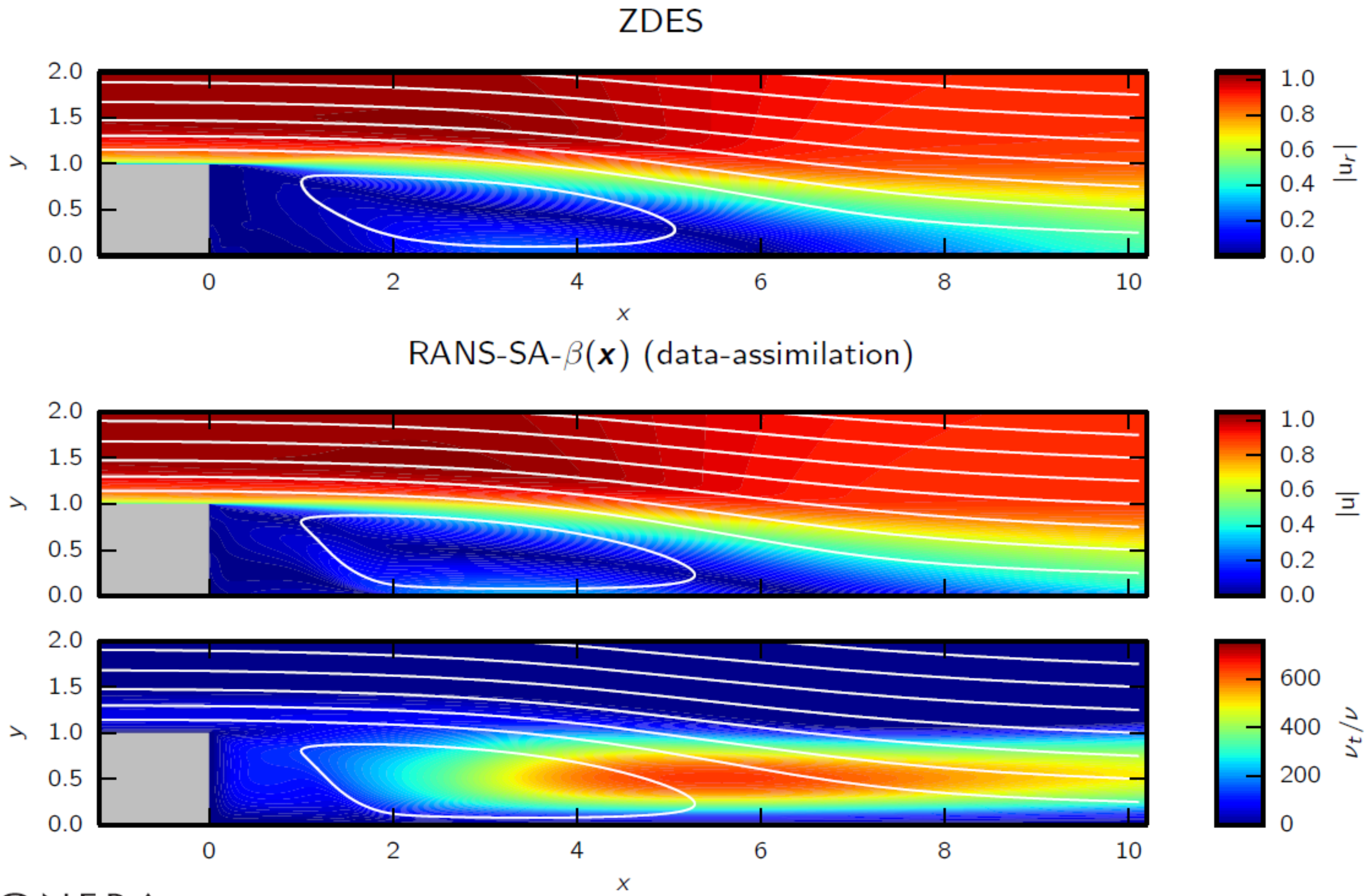
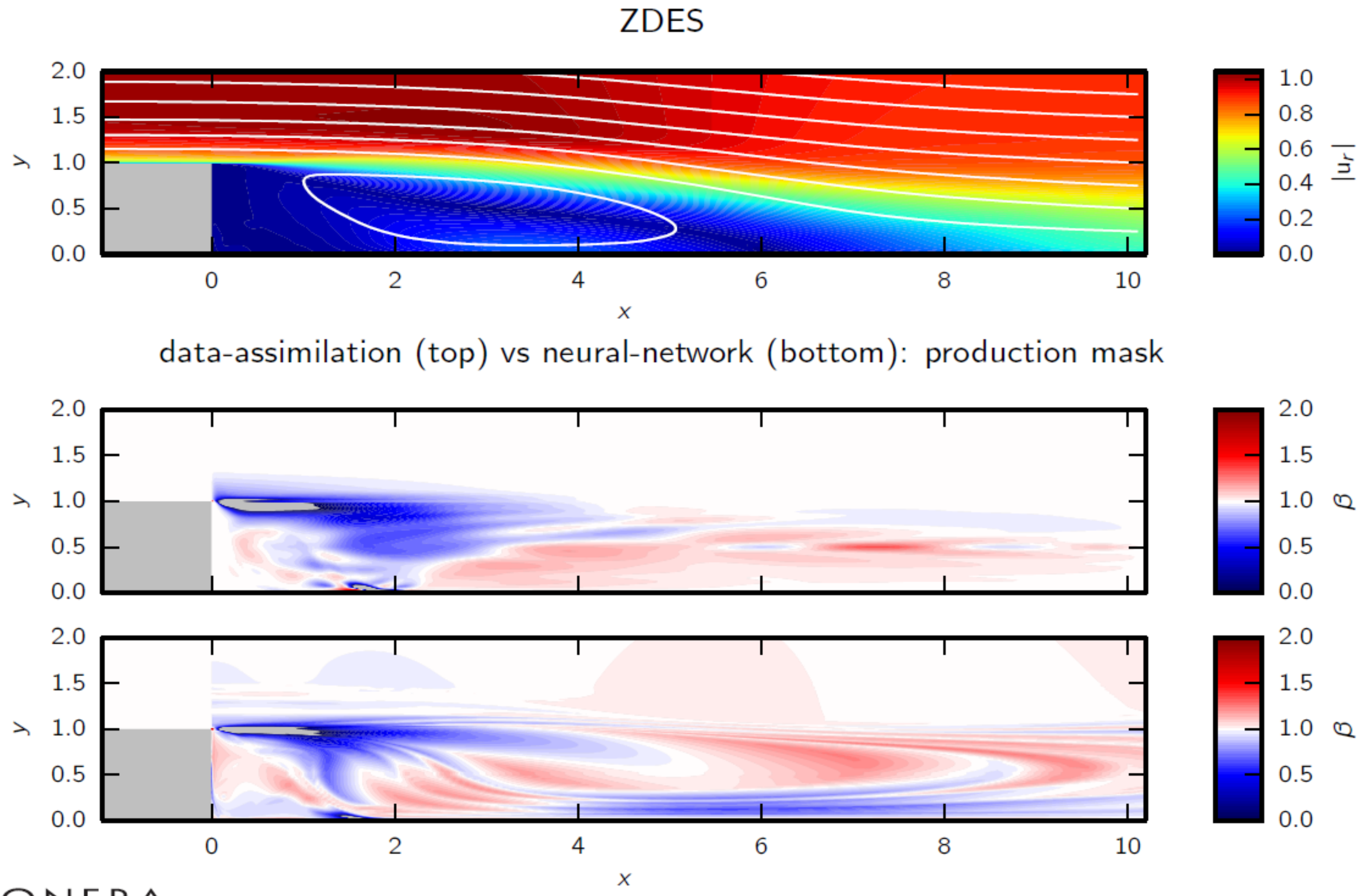
N. Fabbiane: Data-driven turbulence modeling applied to separated flows – 12 of 13

## Validation



ZDES
RANS-SA-$\beta(\boldsymbol{x})$ (data-assimilation)
y
x
$|u_r|$
$|u|$
$\nu_t/\nu$

## Validation


ZDES
y
x
$|u_r|$
data-assimilation (top) vs neural-network (bottom): production mask
β
0 2 4 6 8 10
0.0 0.5 1.0 1.5 2.0

# Data-driven turbulence modeling applied to separated flows
# Conclusion

- A correction of Spalart-Allmaras is fitted to ZDES data.
  - Adjoint-based assimilation algorithm.
  - No appriciable difference between balanced and unbalanced $c_{w1}$.
- A neural network is trained to generalize the identified correction...
- ...and tested on the *training* flow-case.
- The algorithm can be (is already) generalised for different cost functions:
  - pressure matching,
  - skin friction matching
  - ...

  and different measuments techniques, i.e. include a measurment operator in the procedure.

Next step(s):

- Explore different input variables for the neural-network model.
- Blind-test on a *different-but-similar* flow case.

# Turbulence model, HPC & Maching learning
# Conclusion

- Turbulence with DNS at high Reynolds numbers (Scale-Resolved simulation) needs
  - Compute : More than exascale computing resources
  - Huge memory storage
  - Algorithms breakthrough (accuracy, efficiency)
  - Can be mitigated by LES and hybrid RANS/LES

- Turbulence with RANS needs
  - Major improvements in turbulence models
  - Artificial Intelligence combined with increasing capacity of memory storage should allow to make significant steps forward
  - ONERA is developing stronger synergy between CFD, experiments & data

- *European project HiFi-TURB on AI for turbulence modelling starting mid-2019*