Réduction de modèles pour des problèmes d'optimisation et d'identification en calcul de structures

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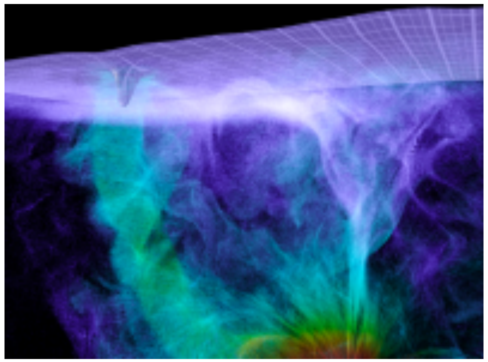

Séminaire Aristote, 8 novembre 2016, École Polytechnique, Palaiseau.

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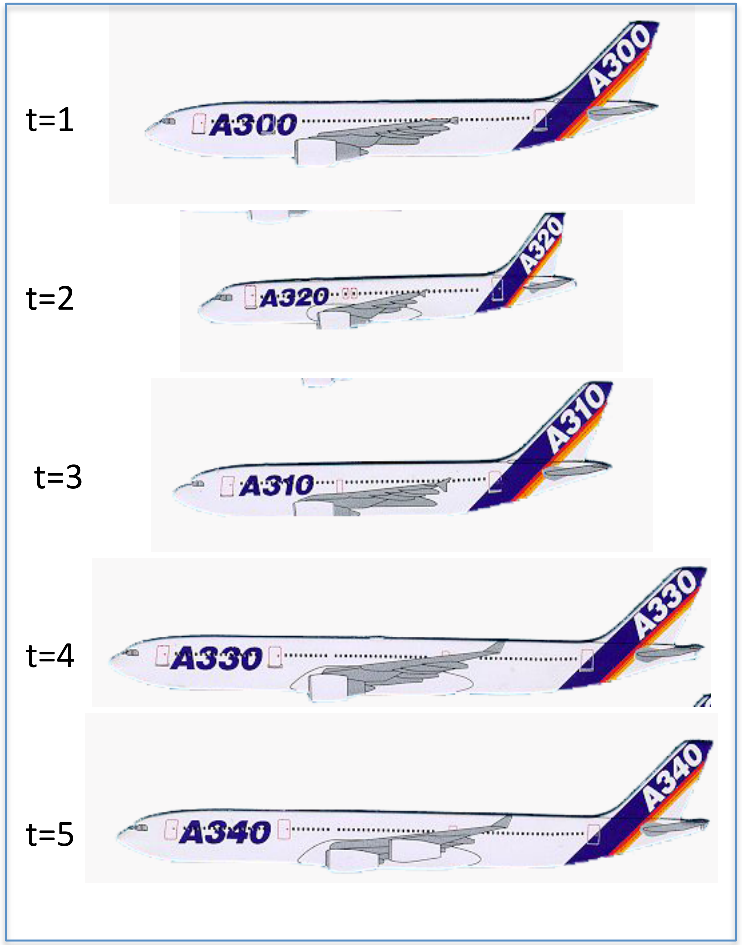
An example of a complex shape to be parameterized

How to find the lowest number of parameters that allow to obtain all the « admissible » shapes ?

Two kinds of problems



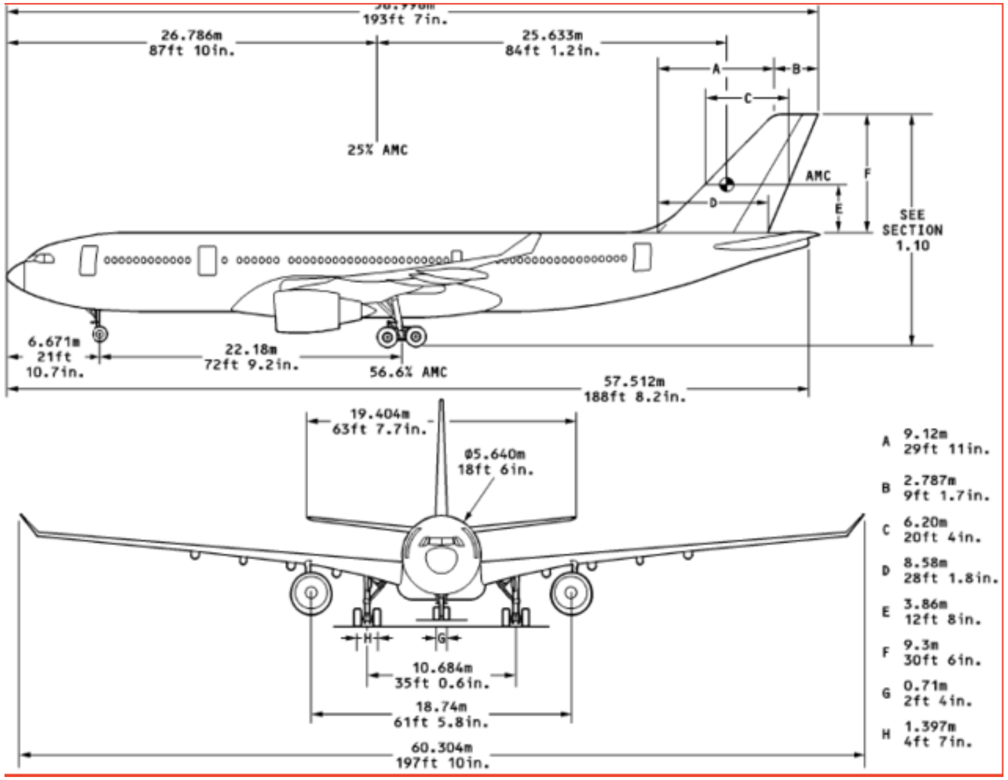
Identification

t=?

interpolation

$t=t_{new}$

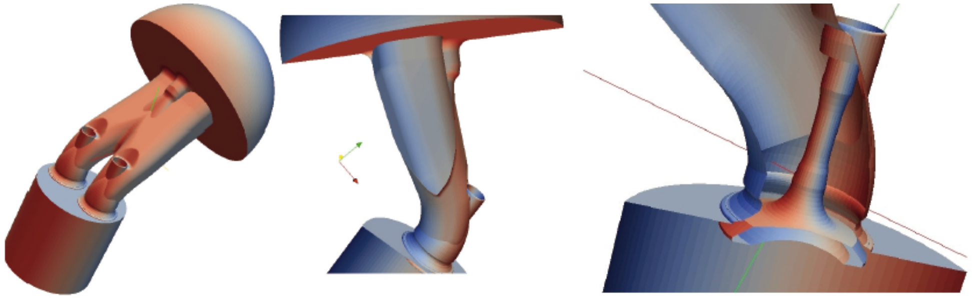
Parameterization issues



- high number of design variables
- implicit technological constraints
- intrinsic dimensionality much lower then the number of variables

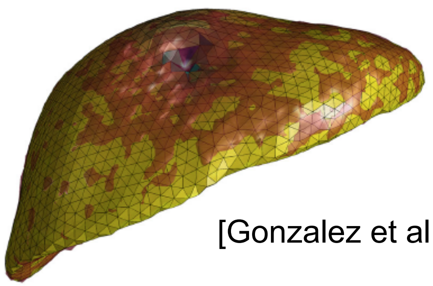
Problems of interest

- Highly dimensional input spaces : reduce the number of parameters (highly constrained CAD)

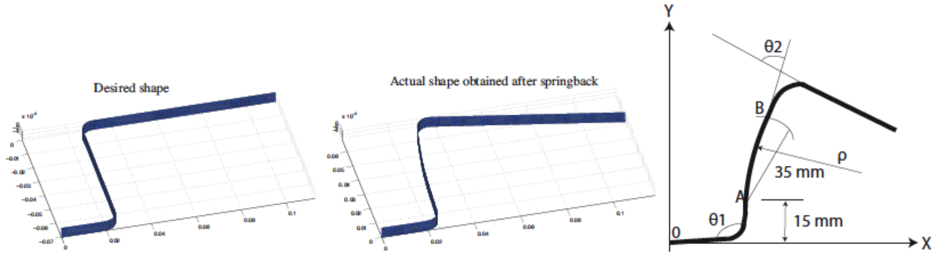


[Raghavan et al., 2013]

- Find the smallest number of parameters for « natural » shapes (biomedical, post-springback)

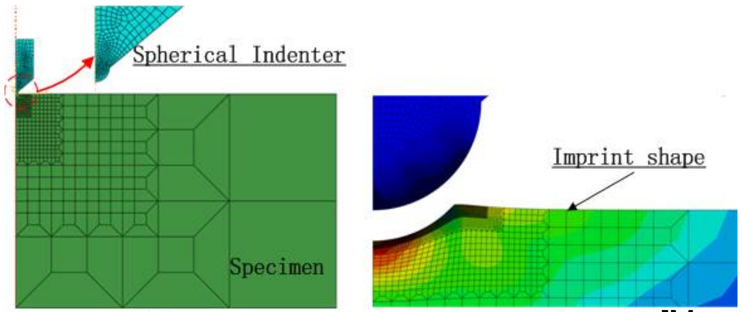


[Gonzalez et al., 2015]

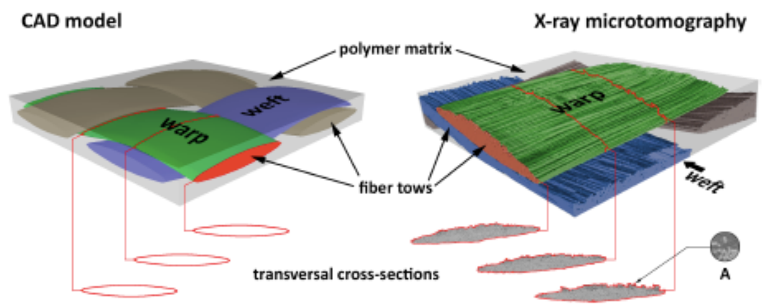


[Le Quilliec et al., 2015]

- Characterize measured shapes (Image correlation)

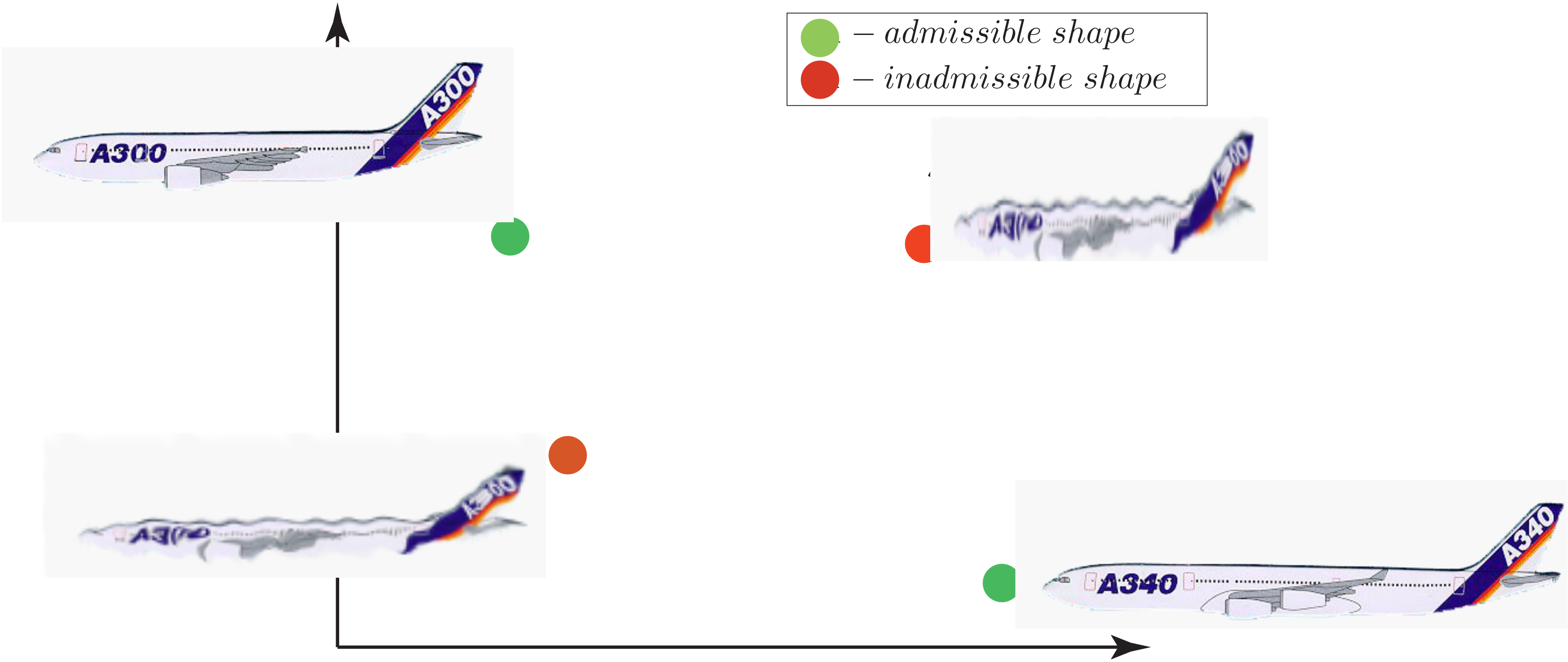


[Meng et al., 2015]



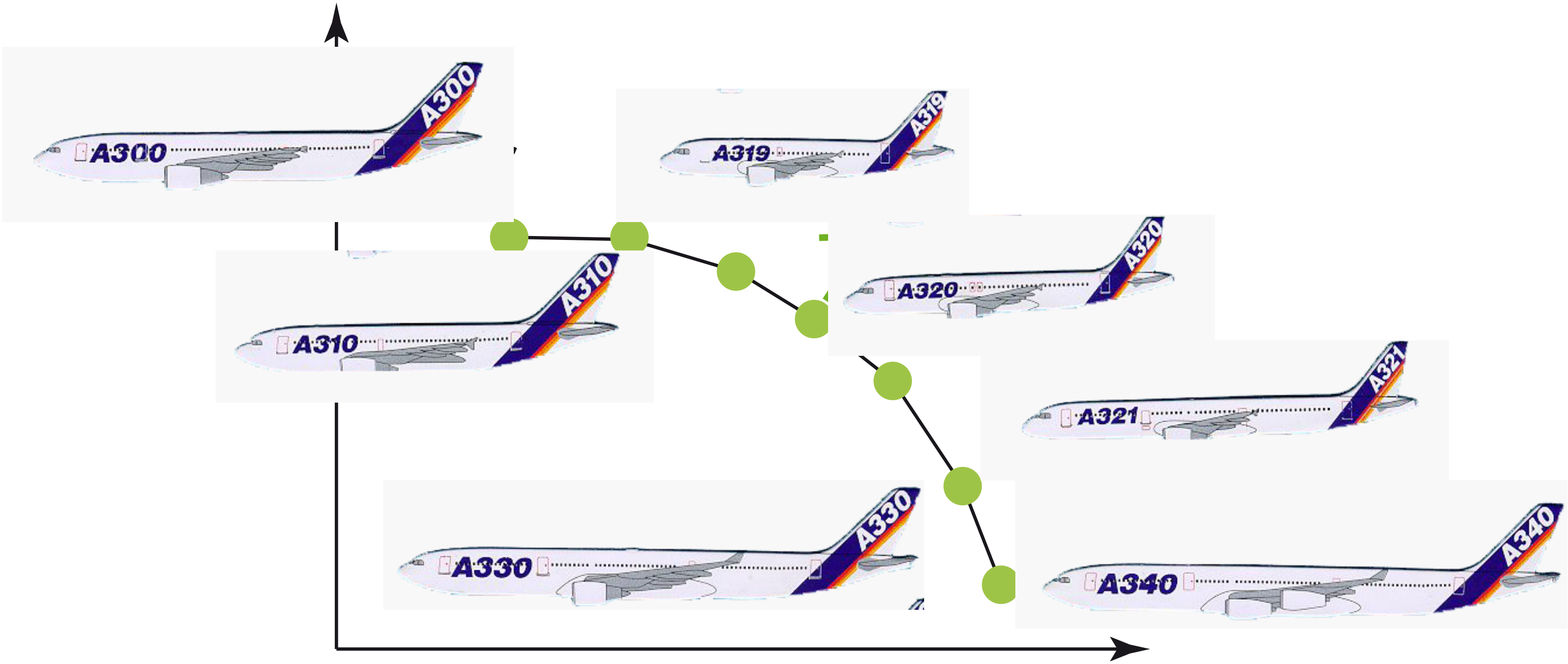
[Madra et al., 2015]

Shape space



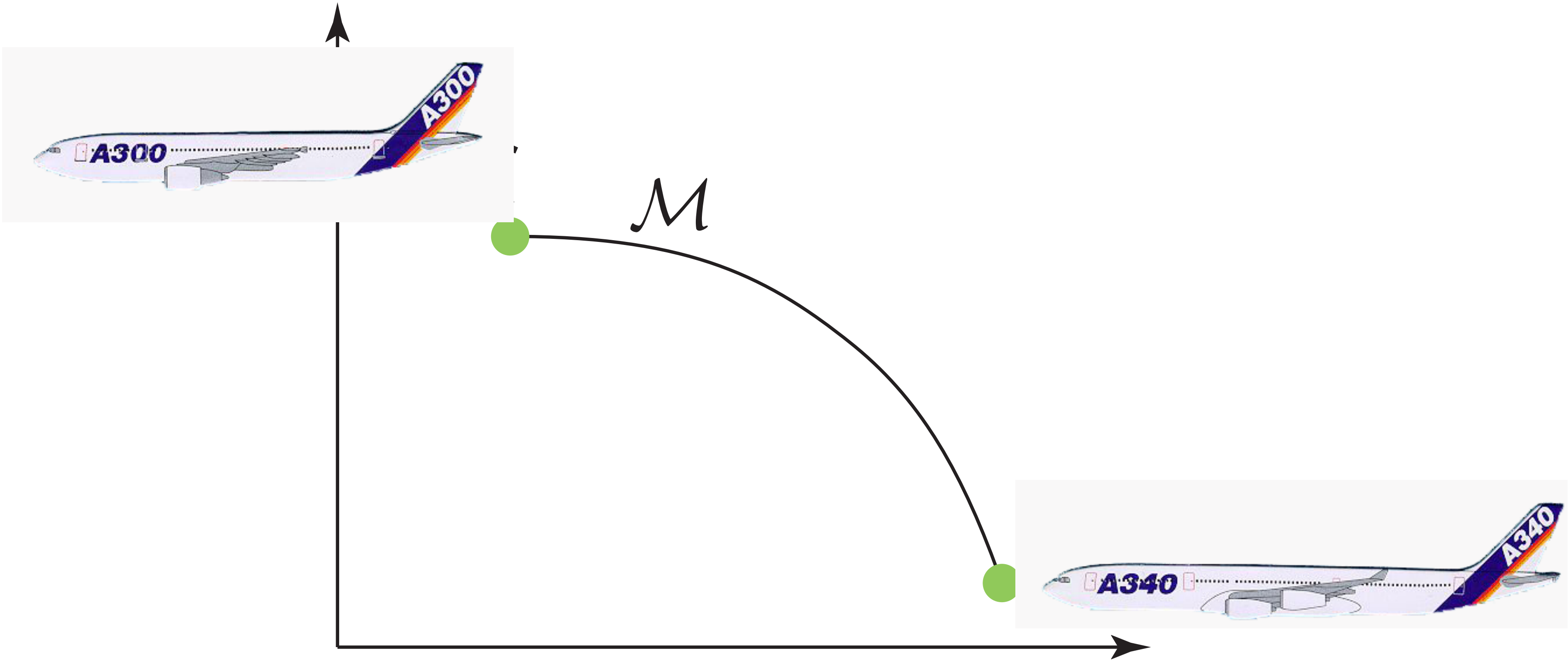
Problem: dimension of the shape space is generally very high

Discrete shape manifold



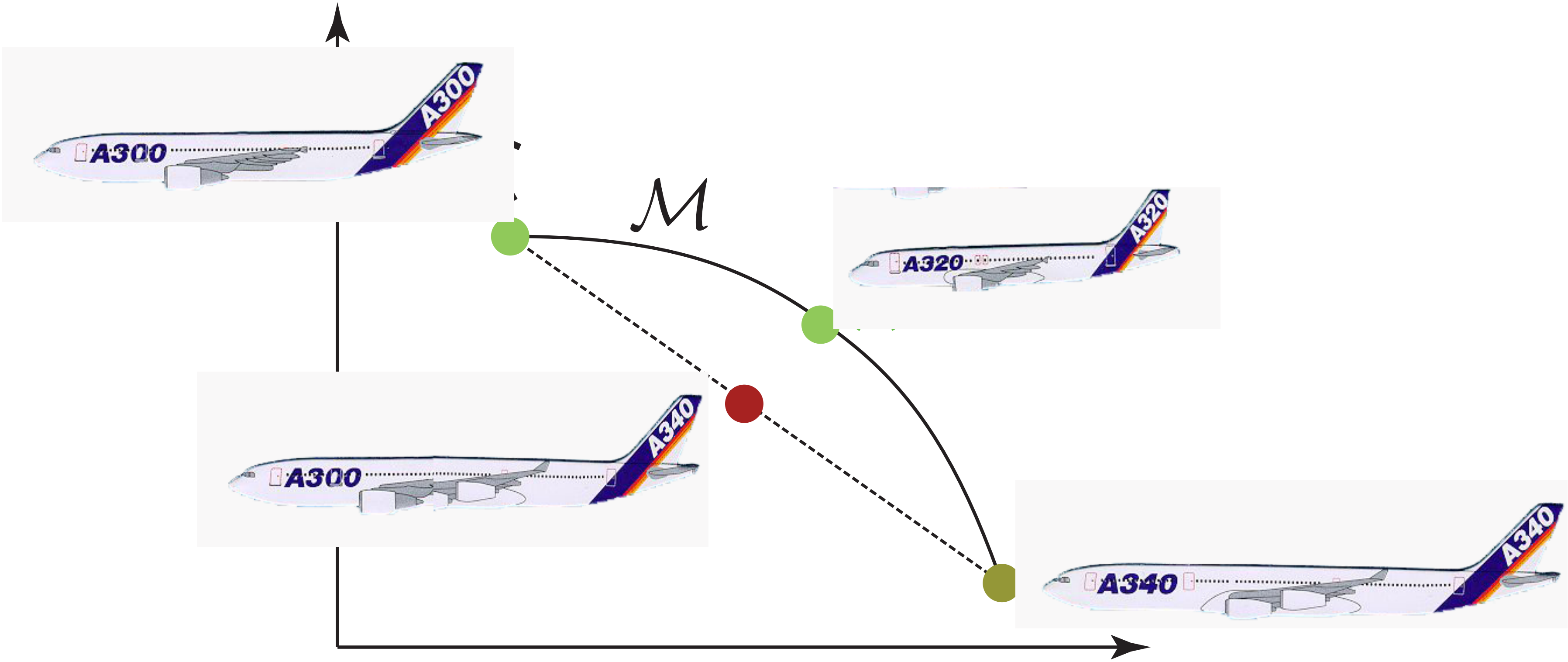
Hypothesis: all admissible shapes belong to a lower-dimensional manifold embedded in shape space

Smooth shape manifold



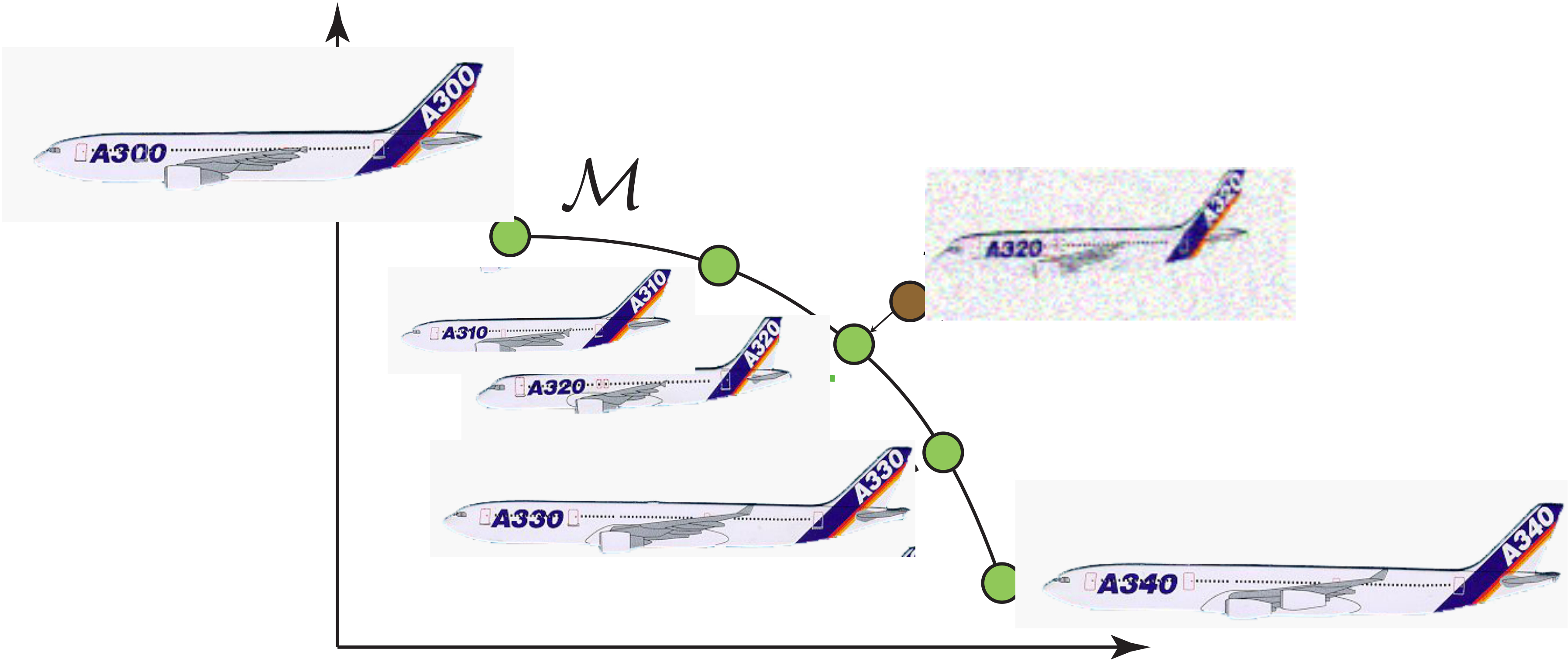
All intermediate shapes may be obtained by interpolation on the manifold

Interpolation in shape space



Cartesian distance has to be replaced by geodesic distance

Projection in shape space



Experimentally measured shapes generally do not belong to the manifold due to the noise

Taxonomy of NLDR methods

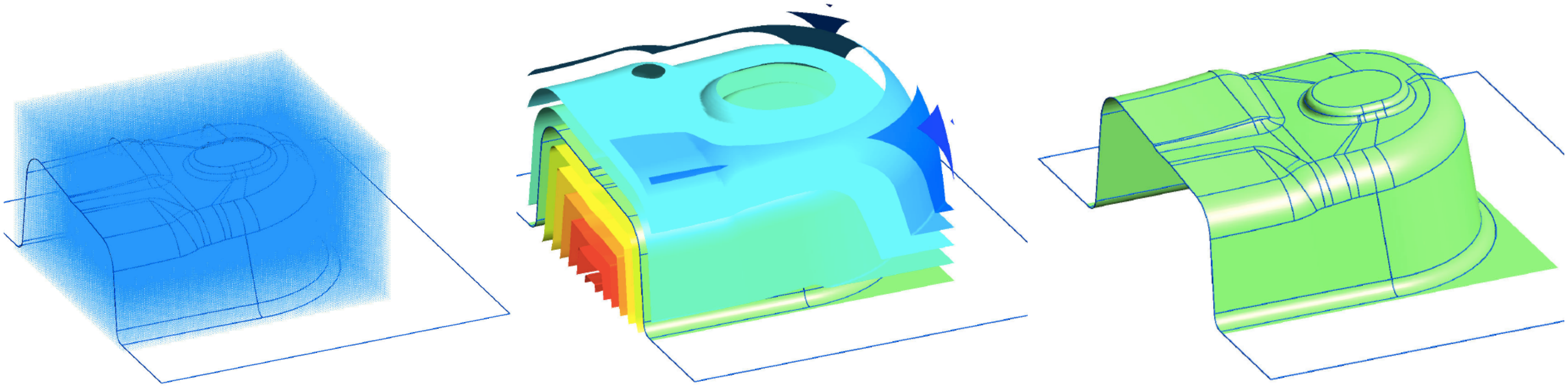
- Proper orthogonal decomposition (POD)
  - covariance matrix of data points, and computes a position for each point.
- Multidimensional Scaling (MDS)
  - matrix of pair-wise distances between all points, and computes a position for each point.
- Isomap (Tenenbaum, de Silva, and Langford) – generalization of MDS
  - assumes that the pair-wise distances are only known between neighboring points > pair-wise geodesic distances
- Locally-Linear Embedding (LLE, Roweis and Saul)
  - find the low-dimensional embedding of points, such that each point is still described with the same linear combination of its neighbors
- Other methods
  - Laplacian eigenmaps
  - kernel PCA
  - Hessian LLE
  - Manifold unfolding
  - ...
- Proposed approach
  - explicit nonlinear manifold construction based on local information
  - intrinsic dimensionality estimation
  - custom-built manifold walking algorithms

Agenda

- Explicit construction of the manifold in reduced shape space
  - admissible shape
  - hypothesis of existence of a smooth manifold
  - intrinsic dimensionality and intrinsic variables
  - Predictor-corrector manifold walking techniques
- Tools
  - Level set representation of structural shapes
  - POD
  - Fukunaga Olsen (also LLE under development)
  - Diffuse Approximation
- Examples
  - shape optimization
  - springback minimization (talk by Guénhaël Le Quilliec)
  - inverse problem (talk by Meng Liang)
  - optimization with categorical variables (talk by Manyu Xiao)

Level set surface representation

$$\varphi(\mathbf{x}) < 0, \mathbf{x} \in \mathbf{\Omega}$$
$$\varphi(\mathbf{x}) = 0, \mathbf{x} \in \mathbf{\Gamma} \cap \mathbf{D}$$
$$\varphi(\mathbf{x}) > 0, \mathbf{x} \notin \mathbf{\Omega}$$



$$\chi(\mathbf{x}(t), t) = 0$$

Proper Orthogonal Decomposition

$$(\chi^1 .. \chi^M)$$

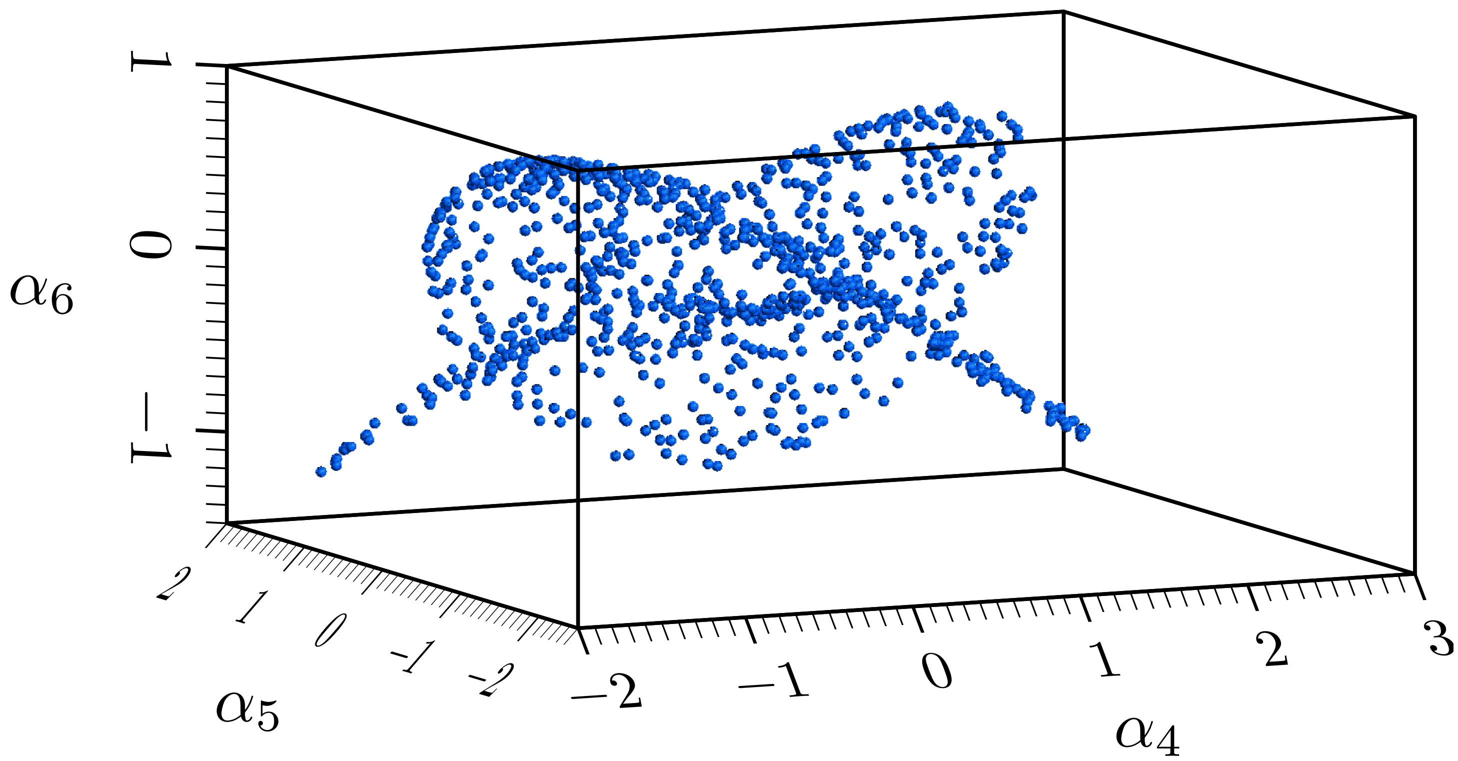
$$D_s = [\chi^1 - \chi_0 \; \chi^2 - \chi_0 \; ... \; \chi^M - \chi_0]$$

$$C_v = D_S . D_S^T$$

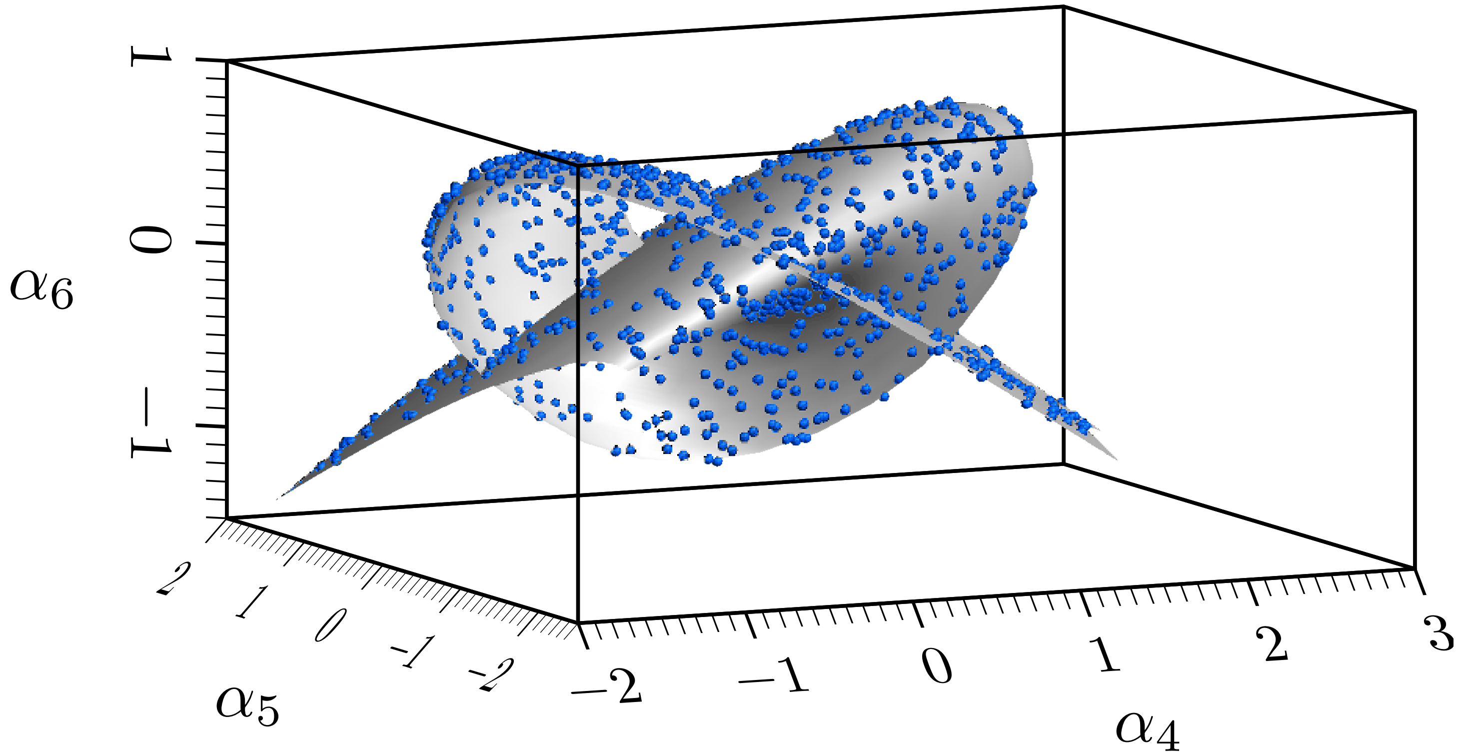
$$\chi^i = \chi_0 + \sum_{j=1}^{M} \alpha_j^i \bar{\phi}_j$$

$$\mu(\alpha^1 .... \alpha^M) = 0$$

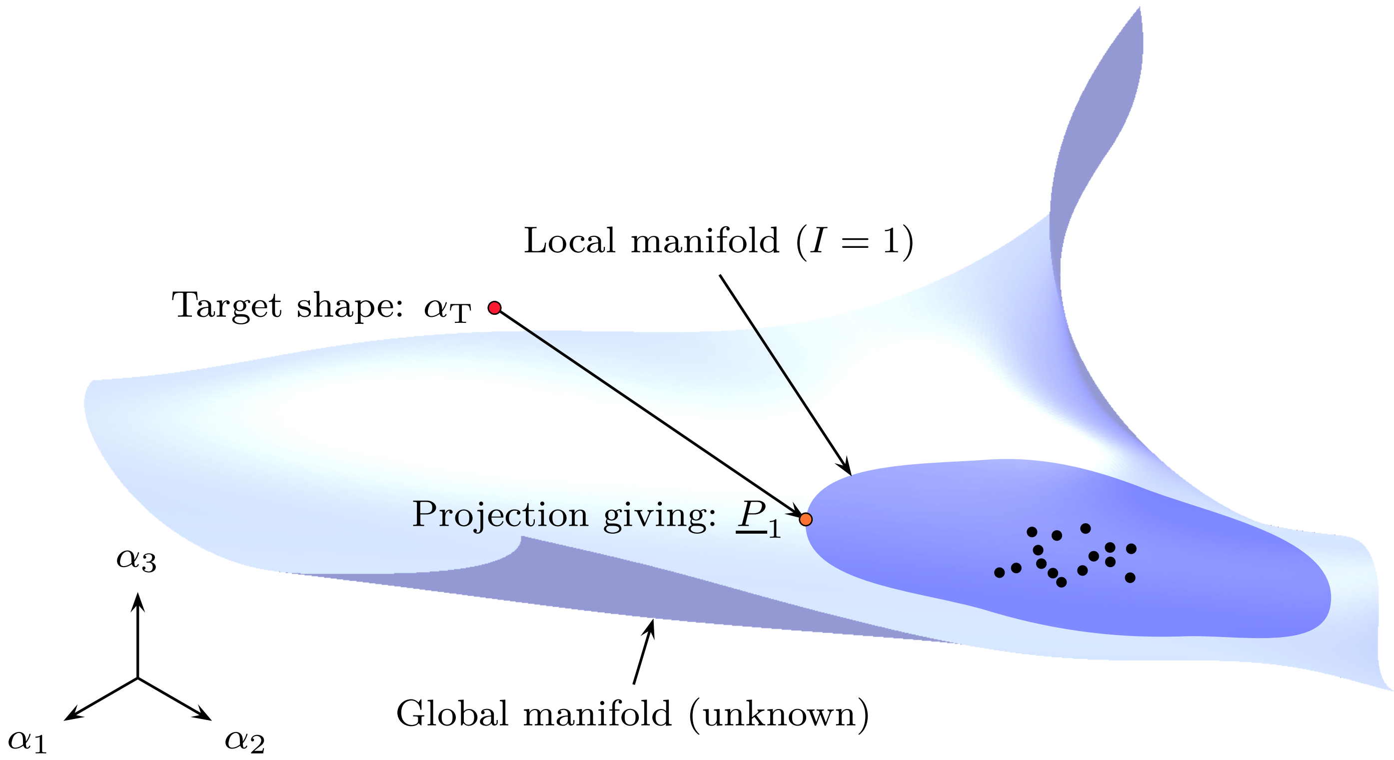
Estimation/detection of dimensionality p



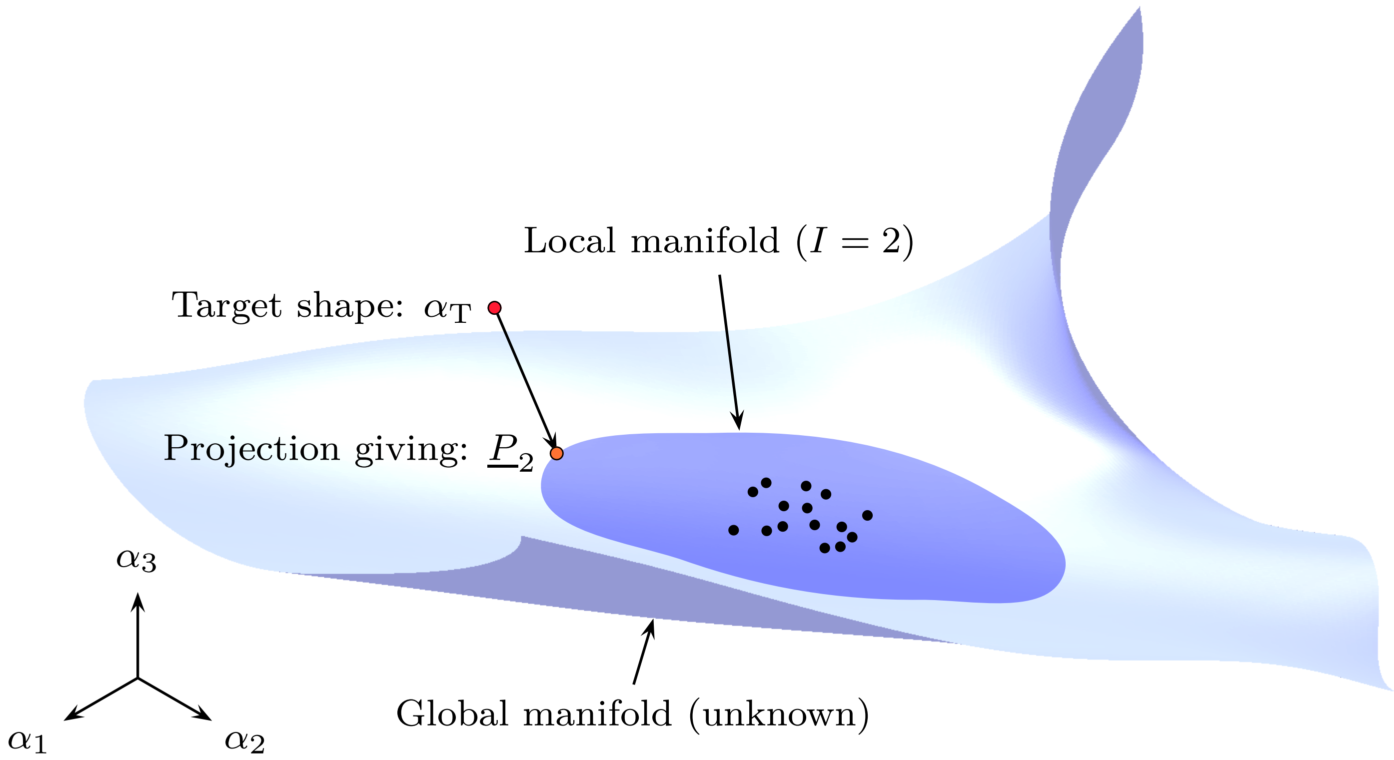
Diffuse Approximation of the manifold



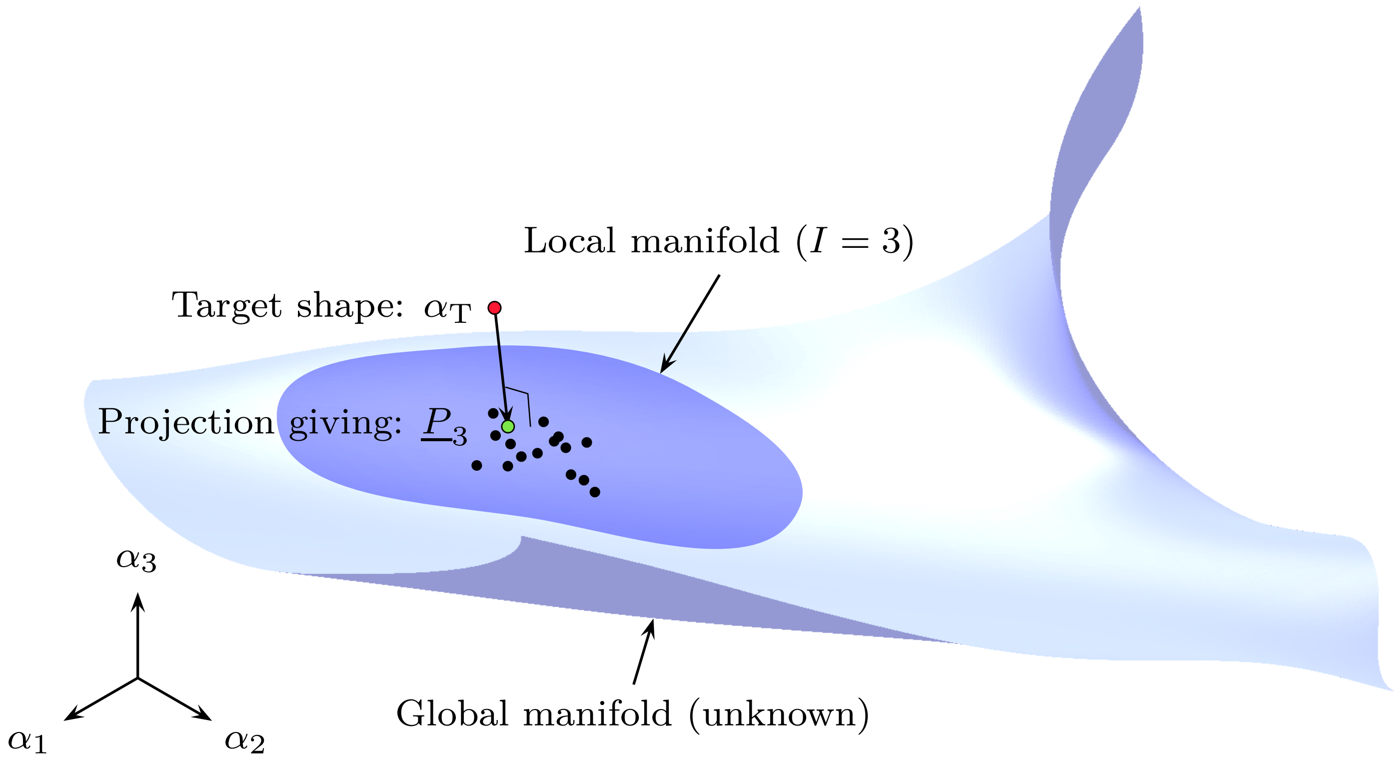
Manifold walking algorithms



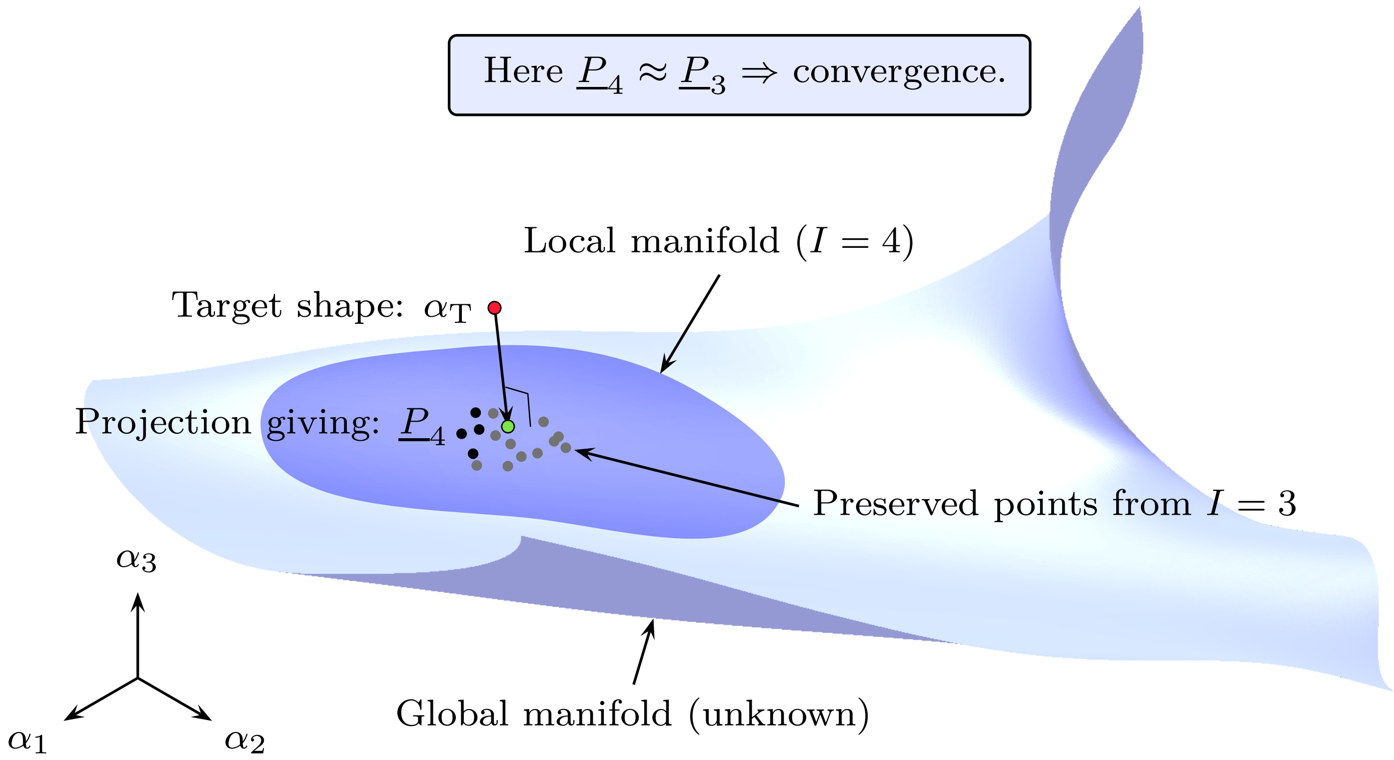
Manifold walking algorithms



Manifold walking algorithms

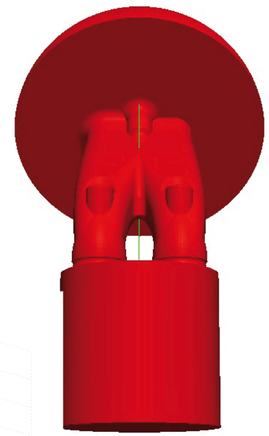


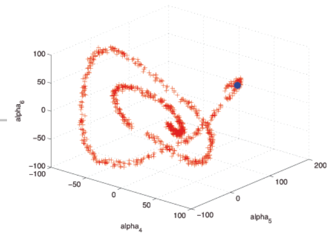
Manifold walking algorithms

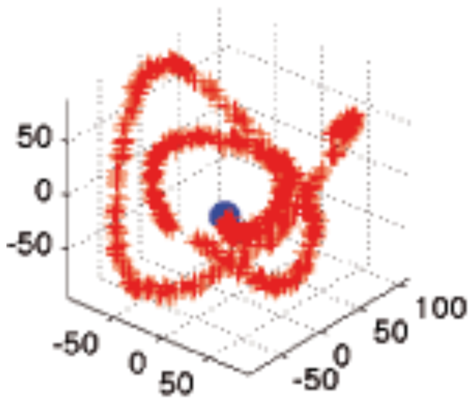


test case 1 : 3D shape optimization

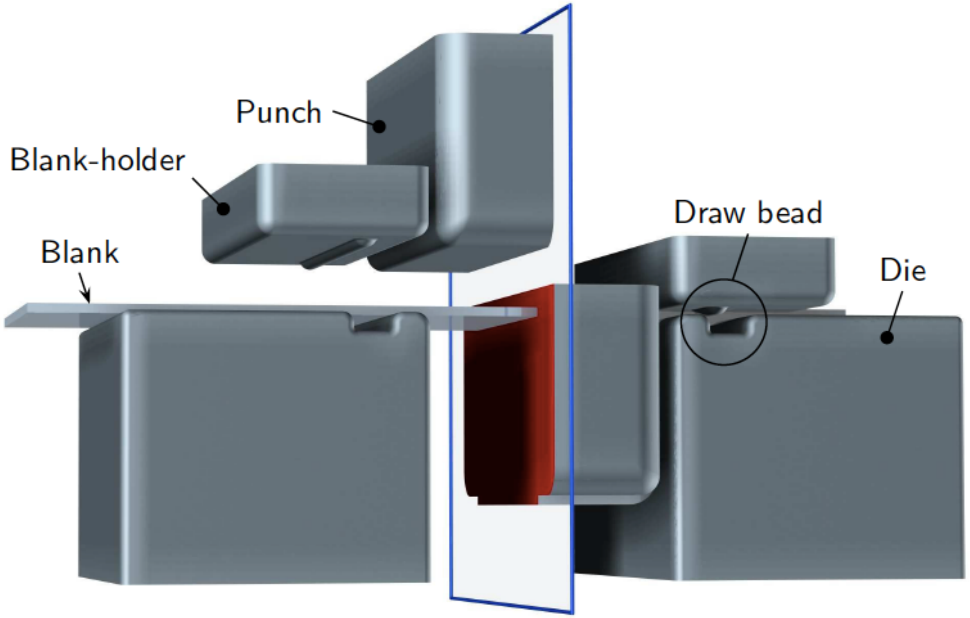
- CAD design
  - 92 parameters
  - unknown number of implicit design constraints
- admissible shape criterion
  - CAD/mesh generator success
  - 8% success rate in DOE

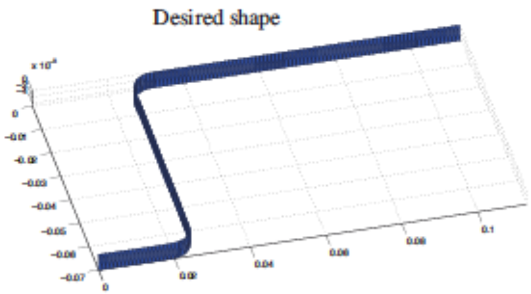


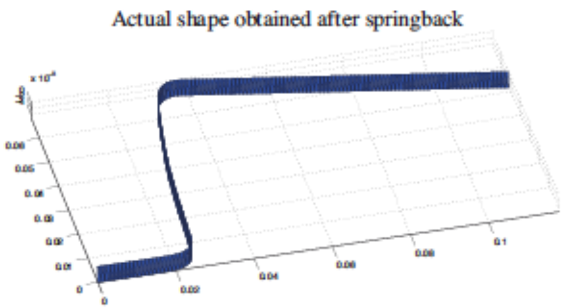


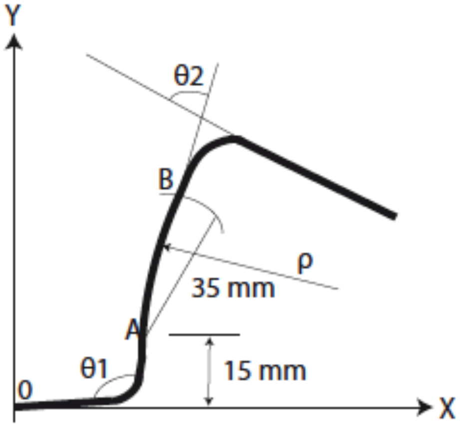


test case 2 : Quantification of springback

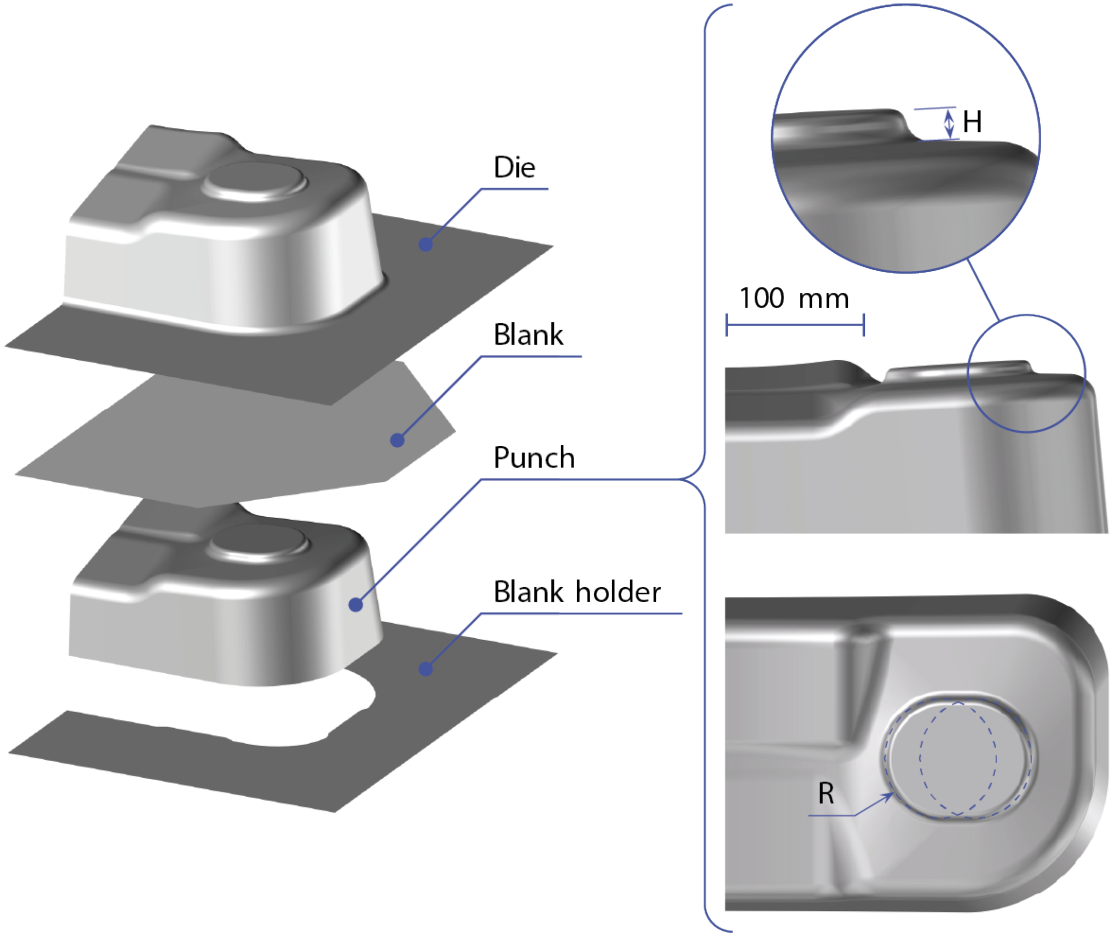




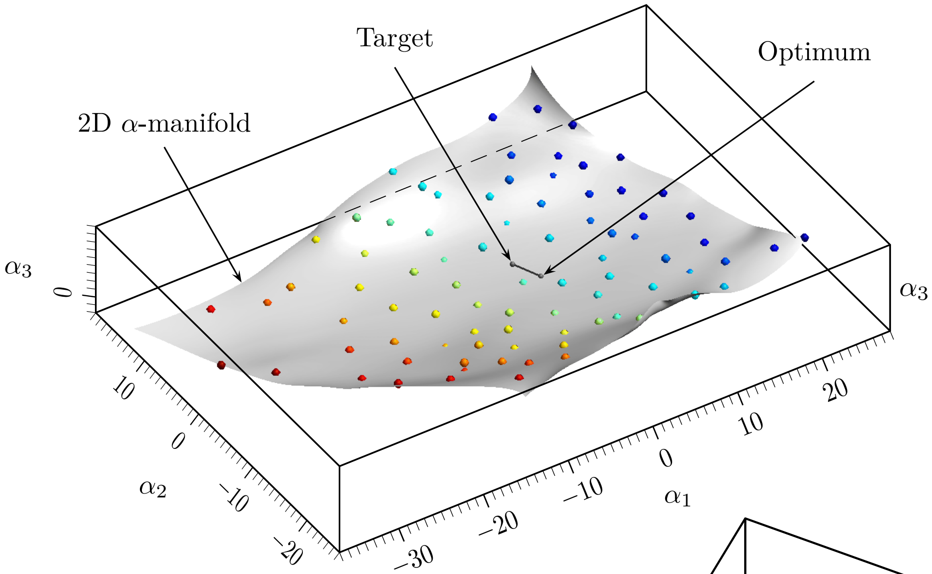


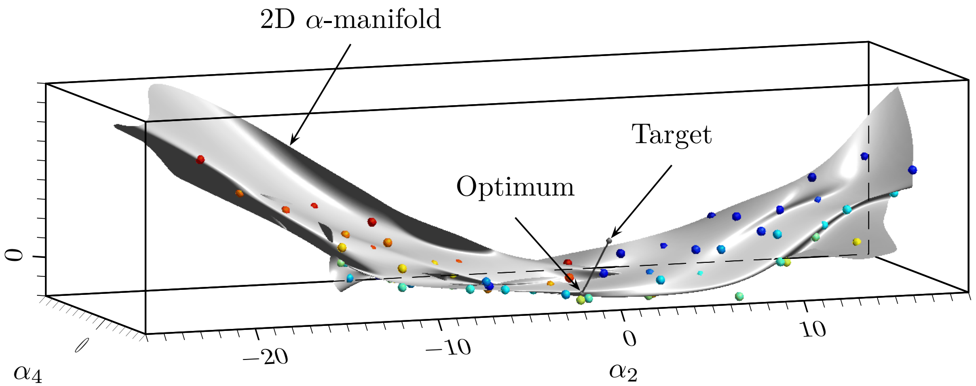


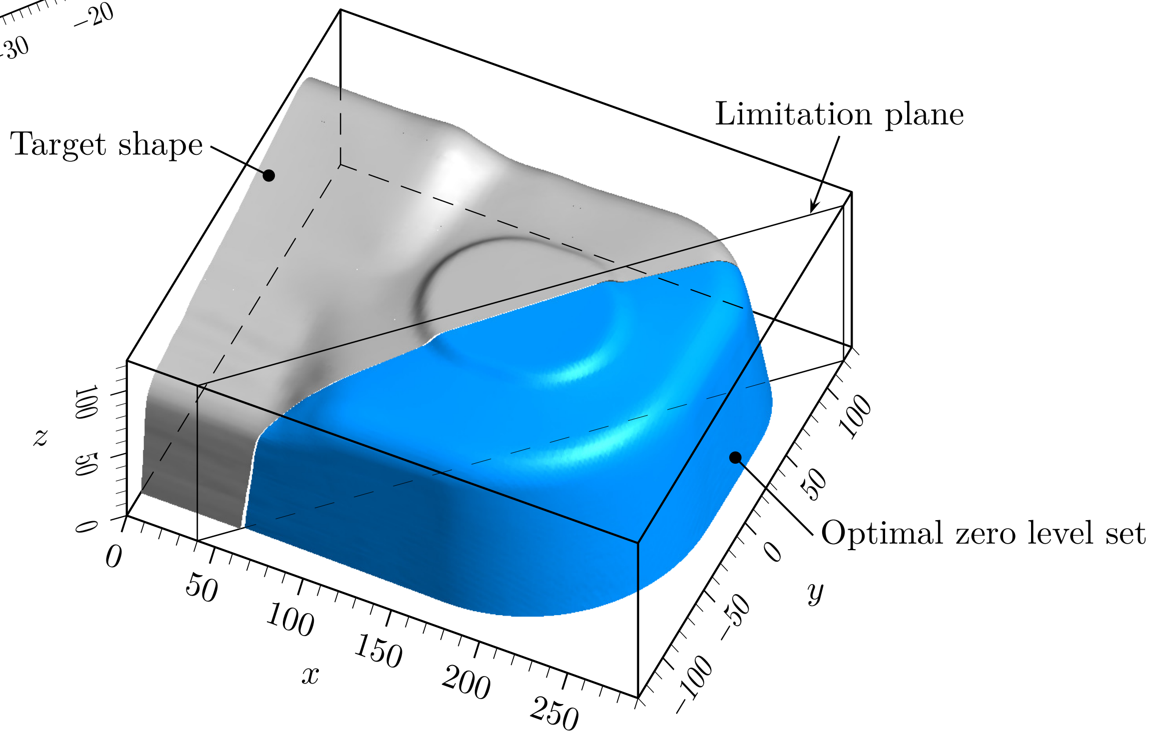
Example on a 3D complex shape: automotive strut tower punch.



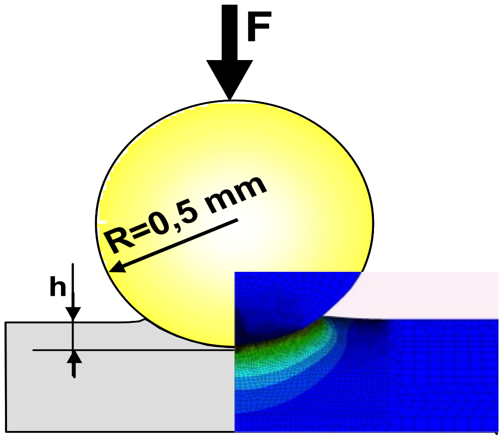
Optimization with 2 tool geometry design variables



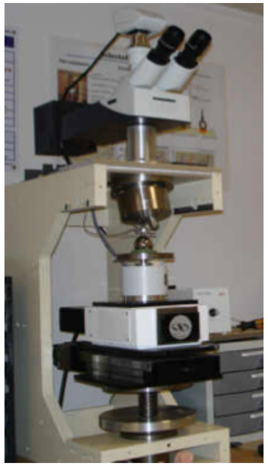




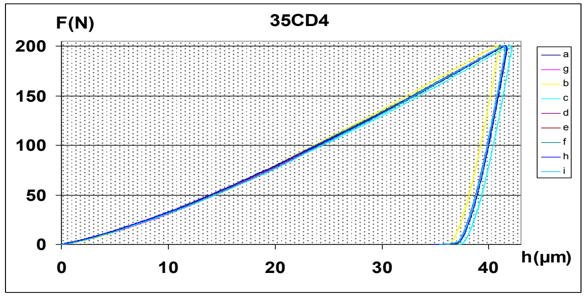
Material characterization by image correlation and inverse analysis
Instrumented indentation test



Indenter & Specimen

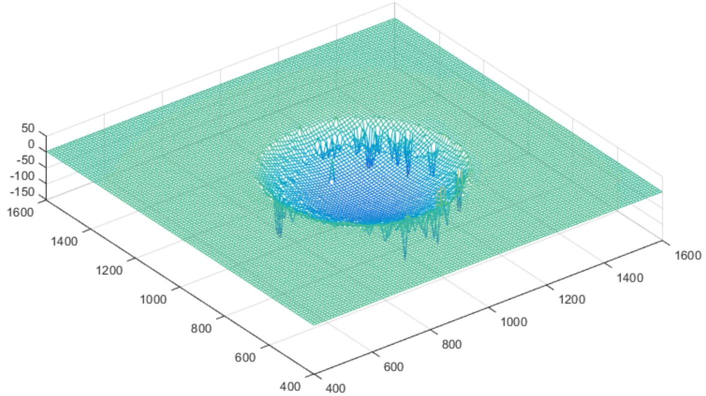


Experimental setup



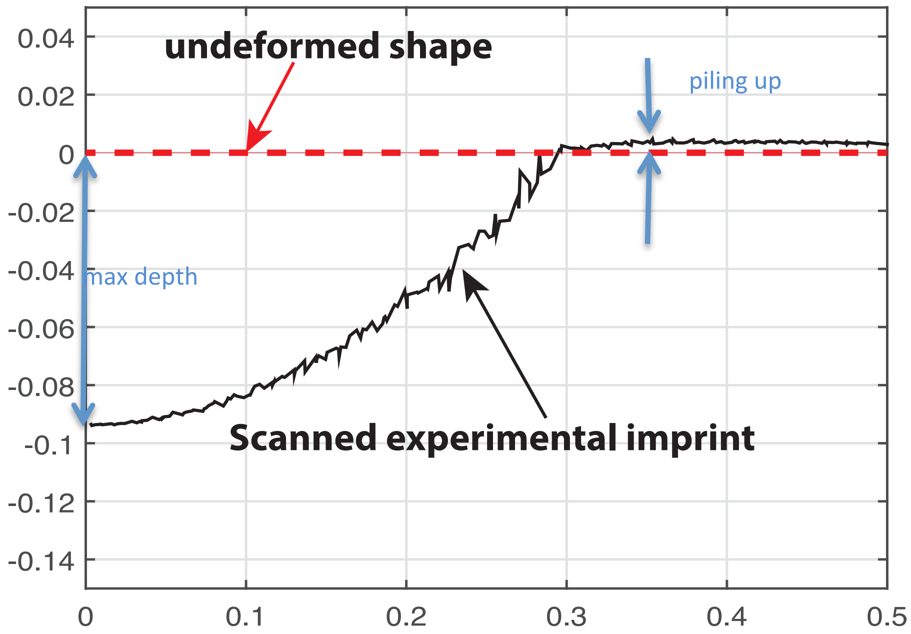
P-h curve

scanning device



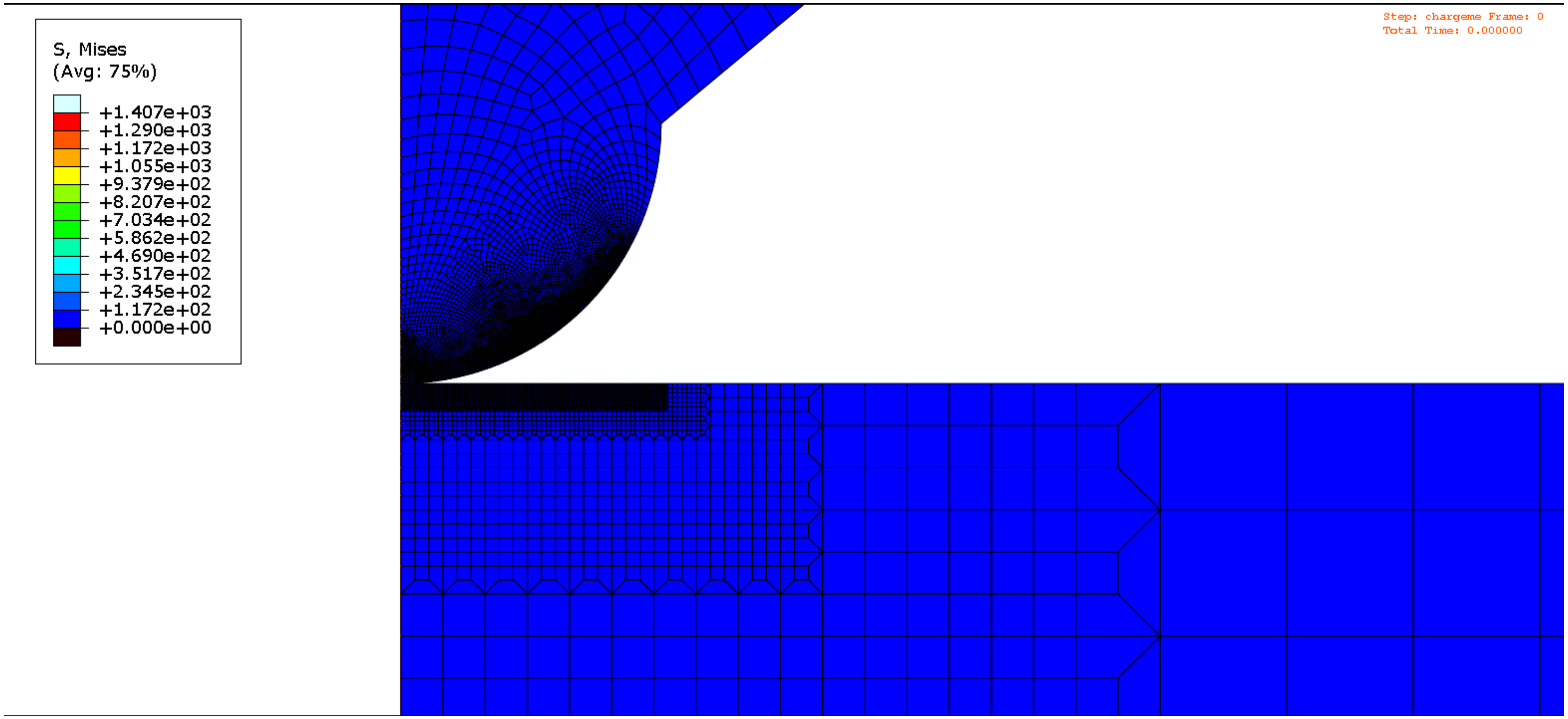
Imprint shape

experimental imprint shape (C100 steel)



- Imprint shape voxelized with resolution 200X200
- Step length of scanning is $10\mu m$;

Finite element simulation



Simulation Parameters :

Coulomb friction coefficient: $\mu = 0.1;$

Maximum force: $F_{\max} = 500N.$

Geometric parameters:

Dimensions of specimen: $59mm \times 59mm$

Radius of Indenter: $r = 0.5mm$

Simulation information:

- Four-node axisymmetric elements(CAX4)
- Elements: 4394 (specimen) and 6070 (indenter)

Postulated constitutive law

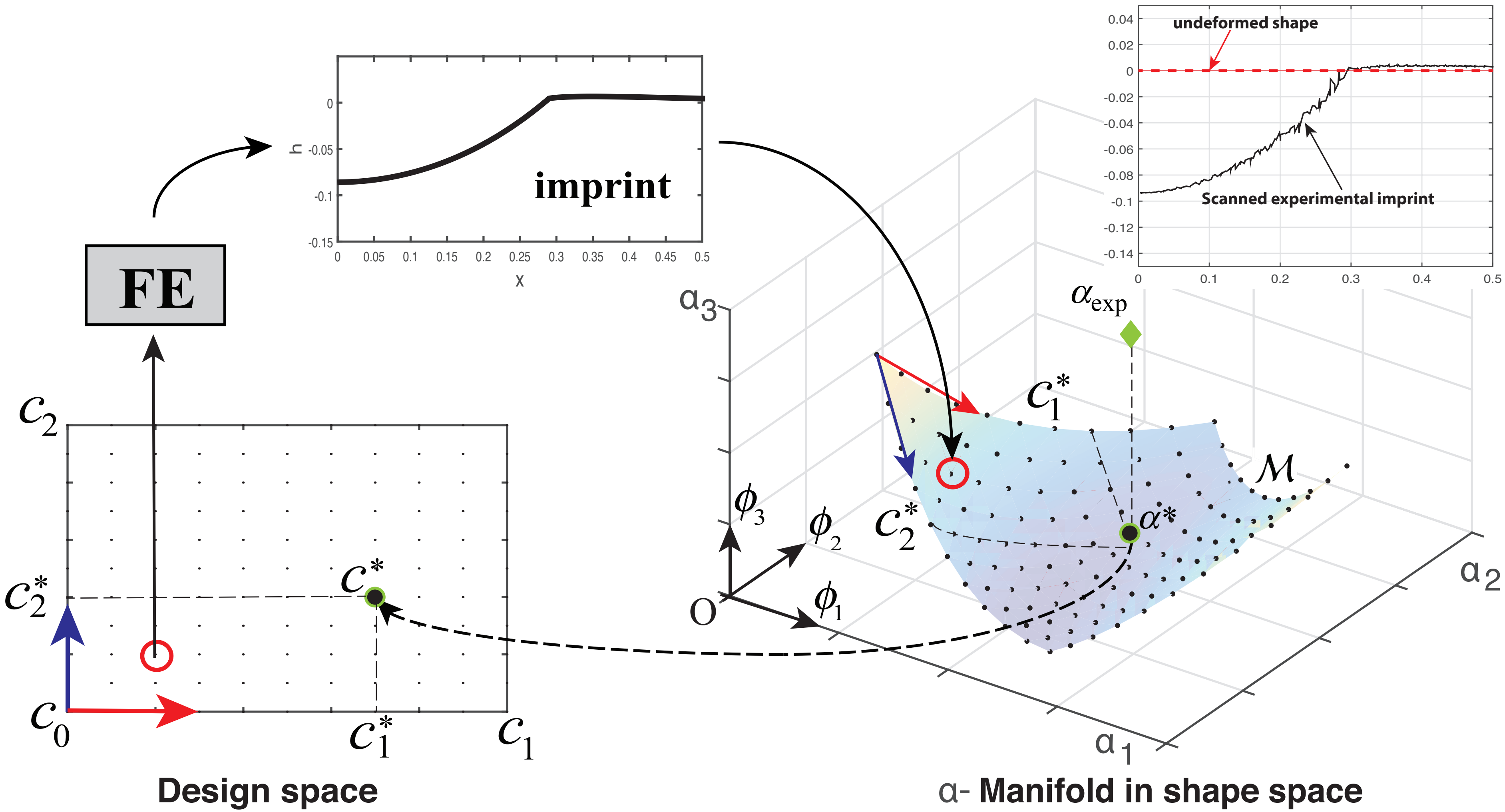
An isotropic Hollomon's law with two parameters

$$\sigma = \sigma_y \left(\frac{E}{\sigma_y} \right)^n \varepsilon^n$$

work-hardening exponent $n \in [0.1, 0.5], \quad \upsilon = 0.3;$

yield stress $\sigma_y \in [50, 400]$ $E = 210GPa;$

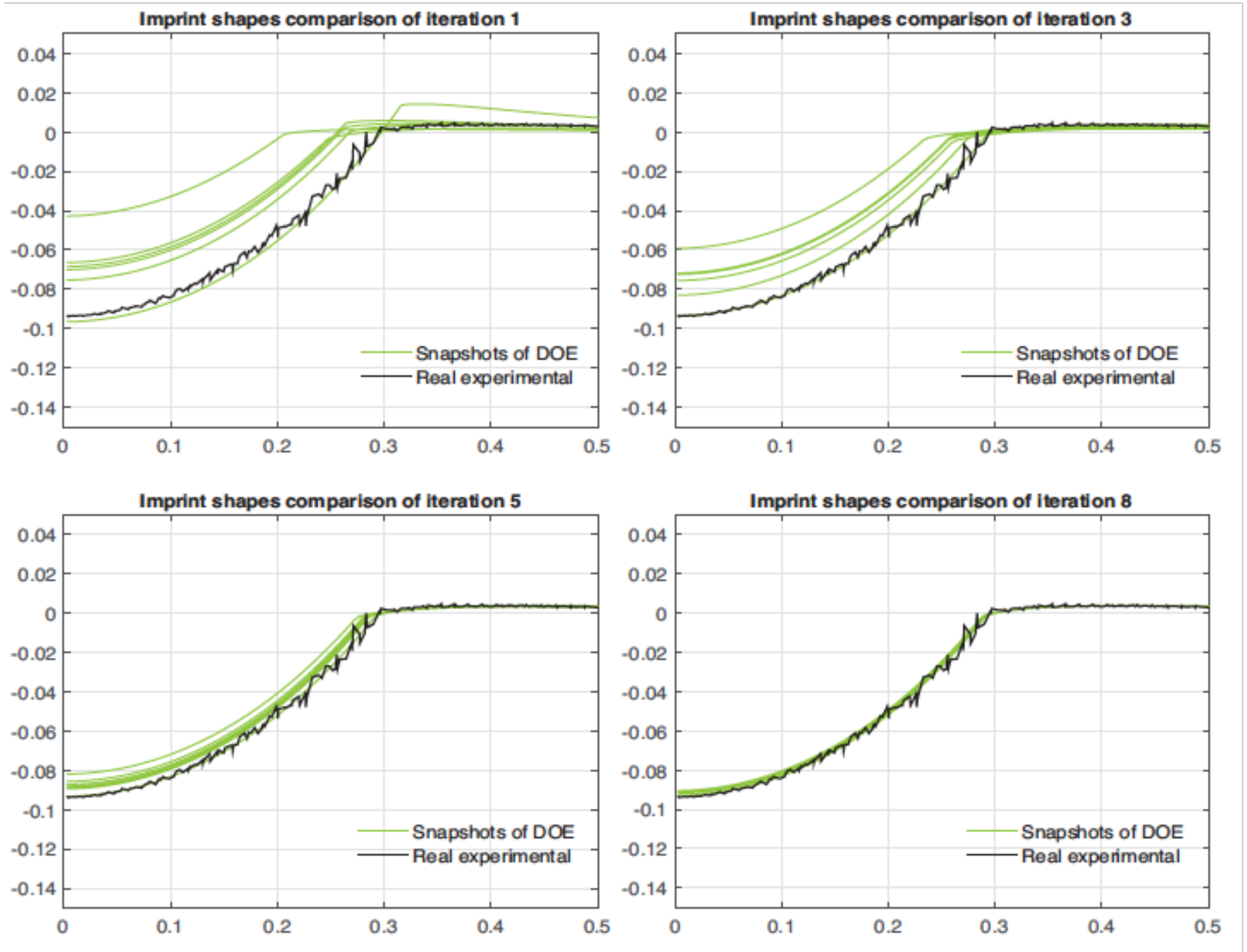
Manifold of admissible shapes



Panning & zooming

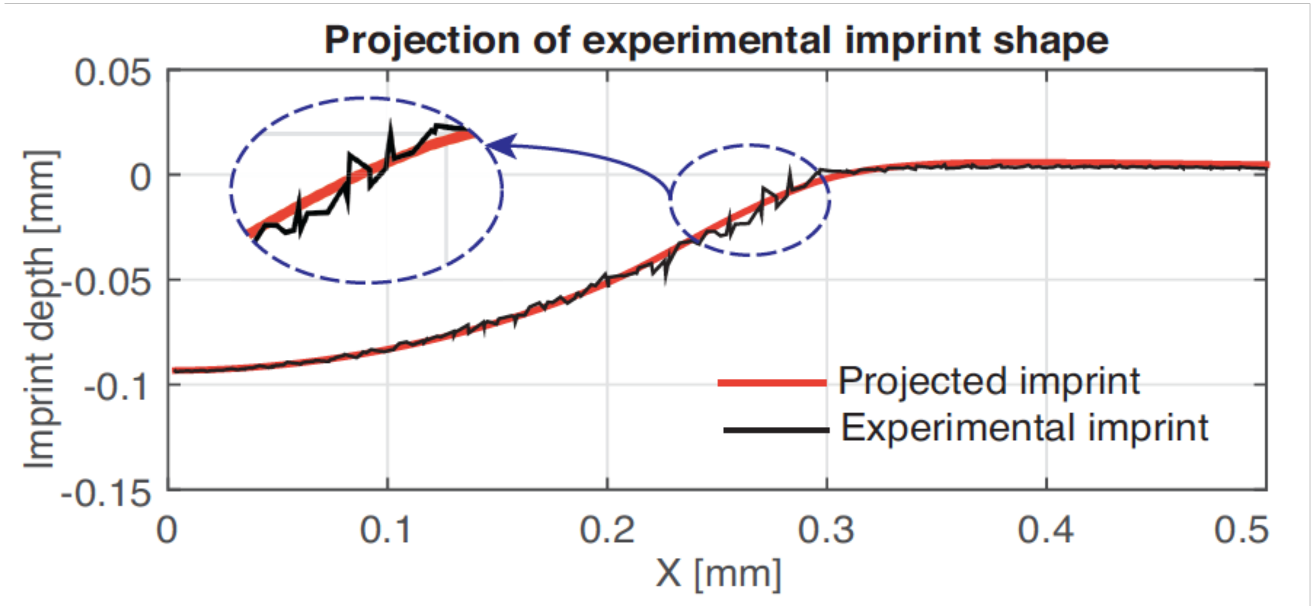
Combination of zooming and panning (Table.3).

Zooming – convergence of snapshots



Experimental imprint and numerical snapshots (zooming algorithm).

Smoothing aspects



Panning & zooming – numerical results

Table 3: Iteration results using panning & zooming.

Iter	σ_y	n	$\Delta\sigma_y$	Δn	$\|\|\boldsymbol{\alpha}_{\text{exp}} - \boldsymbol{\alpha}^*\|\|$	$\frac{\|\|\mathbf{s}_{exp} - \mathbf{s}(\mathbf{c})\|\|}{\|\|\mathbf{s}_{\text{exp}}\|\|}$
1	175.0	0.300	40	0.04	0.1989	25.56%
2	155.0	0.280	40	0.04	0.0697	9.62%
3	135.2	0.287	40	0.04	0.0175	3.98 %
4	120.5	0.307	40	0.04	0.0083	3.68%
5	107.0	0.326	20	0.02	0.0059	3.54%
6	108.0	0.323	10	0.01	0.0121	3.57%
7	106.7	0.323	5	0.005	0.0089	3.56%
8	107.0	0.324	2.5	0.002	0.0096	3.57%

Conclusion and prospects

- A machine learning approach for representation of complex shapes
  - concepts
    - shape space
    - shape manifold
    - intrinsic dimensionality
  - a family of manifold walking algorithms
  - applications
    - metal forming
    - inverse identification problems
    - shape optimization
- Extensions
  - discrete optimization with categorical variables [Gao et al, 2016]
  - Towards simultaneous reduction of both input and output spaces for interactive simulation-based structural design [Raghavan et al. 2013]

Thank you for your attention