Logics for AI:
From Dreams to Formal Methods

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Symbolic AI

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- Traditionally, symbolic AI convenient for settings where the rules are clear cut, and you can easily obtain input and transform it into symbols.
- Logics for AI:
  - formal languages,
  - semantics,
  - proof procedures to reason about it.

Subfields in IJCAI'2023 proceedings

- Agent-based and Multi-agent Systems
- AI Ethics, Trust, Fairness
- Computer Vision
- Constraint Satisfaction and Optimization
- Data Mining
- Game Theory and Economic Paradigms
- Humans and AI
- Knowledge Representation and Reasoning
- Machine Learning
- Multidisciplinary Topics and Applications
- Natural Language Processing
- Planning and Scheduling
- Robotics
- Search
- Uncertainty in AI
- etc.

Subfields in KR'23 proceedings

- Argumentation
- Automated reasoning
- Belief merging / revision
- Conditionals
- Description logics
- Epistemic logic
- Knowledge representation and machine learning
- Multi-agent systems
- Strategic reasoning
- Systems and robotics
- Temporal reasoning

Reasoning on Ontologies

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- Well-known ontologies:
  - Medical ontology SNOMED-CT.
  - NCI Thesaurus (National Cancer Institute, USA).
  - Gene ontology (world largest source of information on the functions of genes).

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- Free ontology editor Protégé

http://protege.stanford.edu/

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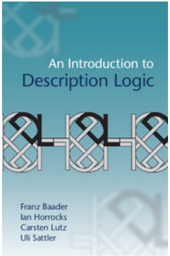
- How to repair faulty ontologies ?
- How to add new concepts or axioms without affecting the old inferences?
- More generally, how to extract from the ontologies more knowledge than what is explicitly specified?
  - inferences about individuals,
  - concept subsumptions, non-redundancy,
  - concept hierarchy, consistency of concepts,
  - etc.

Why Description Logics?



- Formal languages for concepts, relations and instances.
- DLs have all one needs to formalise ontologies.
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  - Acceptable trade-off between expressivity and complexity.
  - Decidability and often tractability.
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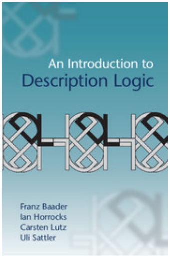

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- A remarkable suite of languages and tools.
  See e.g.,
  - OWL: Web Ontology Language.
  - Protégé: ontology editor.
  - FaCT++: DL reasoner supporting OWL DL.

Description Logic $\mathcal{ALC}$ in a Nutshell

- Language of complex concepts.

$$C ::= \top \mid \bot \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \exists r.C \mid \forall r.C,$$

with concept names A and role names r.

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- Interpretation $\mathcal{I} \stackrel{\text{def}}{=} (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$
  - $\Delta^{\mathcal{I}}$: non-empty set (the **domain**).
  - $\cdot^{\mathcal{I}}$: **interpretation function** such that

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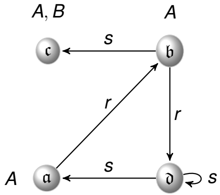

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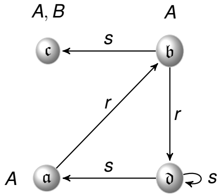

- $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ defined inductively providing the semantics to complex concepts.

Concept Inclusions and Decision Problems

- General concept inclusions $C \sqsubseteq D$ (GCIs).

  E.g., $\texttt{Employee} \sqsubseteq \exists \texttt{WorksFor}.\top$

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- Terminological Box (TBox) $\mathcal{T}$: finite set of GCIs.
- Interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, TBox $\mathcal{T}$.

$$\mathcal{I} \models \mathcal{T} \quad \overset{\text{def}}{\Leftrightarrow} \quad \text{for all } C \sqsubseteq D \in \mathcal{T},\ \mathcal{I} \models C \sqsubseteq D$$

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- **Concept satisfiability problem w.r.t. general TBoxes**:

  Input: A concept C_0 and a TBox $\mathcal{T}$.

  Question: Is there an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $C_0^{\mathcal{I}} \neq \emptyset$?

- This problem is ExpTime-complete.

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- General scheme for integrating concrete domains in DLs.
  [Baader & Hanschke, IJCAI'91]
  - declarative semantics close to the usual semantics for DLs,
  - generic extensions of DLs with various concrete domains,
  - tableaux-based algorithms combined with theory reasoning.

Methods for Handling Concrete Domains

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- Translation into automata-based problems.
  – Concept satisfiability problem w.r.t. general TBoxes for $\mathcal{ALC}(\mathbb{N}, <, =, (=_n)_{n \in \mathbb{N}})$ in EXPTIME.
  [Labai & Ortiz & Šimkus, KR'20]
  – Concept satisfiability problem w.r.t. general TBoxes for $\mathcal{ALC}(\{0, 1\}^*, <_{\text{pre}})$ in EXPTIME. [Demri & Quaas, JELIA'23]

(constraint automata for data words or data tree

Temporal Logics with Concrete Domains

- Concrete domains in TCS:
  - Satisfiability Modulo Theory (SMT) solvers.
    String theories, arithmetical theories, array theories, etc.
  - Verification of database-driven systems.
  - Temporal logics with arithmetical constraints.

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- "Infinitely often $\mathtt{x}$ is a prefix of the next value for $\mathtt{y}$":
  $\mathsf{GF}(\mathtt{x} <_{\mathsf{pre}} \mathsf{X}\mathtt{y})$.
- Satisfiability problem for $\mathsf{CTL}^*(\mathbb{Z}, <, =, (=_n)_{n \in \mathbb{Z}})$ is decidable in 2EXPTIME.

  [Carapelle et al, JCSS 2016; Demri & Quaas, CONCUR'23]

(CTL*: *well-known logic related to model-checking*)

Another Success Story: Logics of Strategic Ability

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- Well-known specimens.
  - Coalition Logic CL. (one-step strategies)
  - Alternating-time temporal logic ATL. (generalisation of temporal logics)
  - Strategy Logic SL. (explicit quantification over strategies)

(with a huge amount of variant

Multi-Agents Systems

- Multi-agent systems are transition systems in which transitions are fired when simultaneous actions are performed by different agents.

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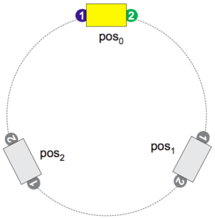

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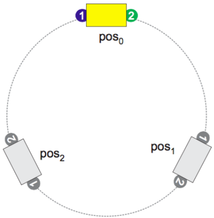

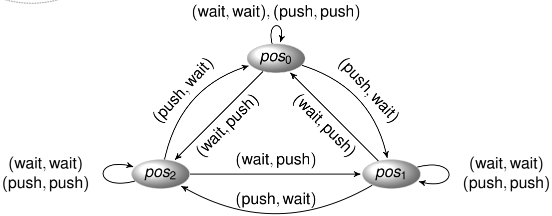

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  $\exists$ strategy σ such that
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- Tractable model-checking problems and automata-based satisfiability-checking decision procedures.

More Ingredients

- Resource-aware logics:
  - actions have costs/weights,
  - formulae may specify constraints about such (cumulative) costs/weights.

  E.g. [Belardinelli & Demri, AI 2021; Bulling & Goranko, AAMAS 2022]

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- Strategic reasoning with knowledge operators.

  See e.g. [Agotnes, Synthese 2006]

- Restriction on agents' knowledge.
  - Strategy logics with imperfect information.
  - Undecidability can be obtained easily.
  - Fragments including those with hierarchies of knowledge leads to less expensive reasoning tasks.

  See e.g. [Berthon et al, TOCL 2021]

In My Reading List

- Well-identified potential interactions between machine learning and symbolic AI.
  - Machine learning can be used to solve logical problems and to accelerate verification/automated techniques.
  - Logical methods can be used to complement learning algorithms to improve the precision and explanability.

(hybridization?)

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- An SMT-Based approach for verifying binarized neural networks [Amir et al., TACAS'21]

(SMT-based approach for formal verification of binarized neural networks)

Logics for AI: the Great Return ?

- Beyond knowledge representation and reasoning for description logics, strategy logics, dynamic epistemic logics, etc. ? *(new applications?)*

- Arithmetical theories for the verification of neural networks. *(new arithmetical theorie*

- Learning logical formulae and unique characterisations. *(new logical problems?)*

A Selection of Bibliographical References

Logical Formalisms with Concrete Domains

- "Description Logics with Concrete Domains—A Survey"

  [Lutz, AiML'02]

  (a classical paper on dc domains)

- "Concrete domains in logics: a survey"

  [Demri & Quaas, SIGLOG News 2021]

  (a brief survey)

- "Using Model Theory to Find Decidable and Tractable Description Logics with Concrete Domains"

  [Baader & Rydval, JAR 2022]

  (model theory for DL

Strategic Reasoning and Resources

- "Alternating-time temporal logic"

  [Alur & Henzinger & Kupferman, JACM 2022]

  (the classical paper about ATL)

- "Combining quantitative and qualitative reasoning in concurrent multi-player games"

  [Bulling & Goranko, AAMAS 2022]

  (how to mix quantitative and qualitative ob

- "Strategic reasoning with a bounded number of resources: The quest for tractability" [Belardinelli & Demri, AI 2021]

  (complexity analy ATL)

Learning and Modal/Temporal Logics

- "Scalable Anytime Algorithms for Learning Fragments of Linear Temporal Logic" [Raha et al., JOSS 2024]

  (how to learn temporal formulae)

- Unique characterisability and learnability of temporal instance queries [Fortin et al., KR'22]

  (Example set (E^+, E^-) to characterise temporal formulae)

- Logic of "Black Box" classifier systems [Liu & Lorini, WoLLIC'22]

  (product modal logic for multi-classifier model

Logic, Verification and Neural Networks

- "Neural Network Verification with Proof Production"
  [Isac et al., FMCAD'22]
  (how to add proof production capabilitie
- "An SMT-Based approach for verifying binarized neural networks" [Amir et al., TACAS'21]
  (SMT-based approach for formal verification of binarized neural networks)
- "Simplifying neural networks using formal verification"
  [Gokulanathan et al., NFM'20]
  (how to remove component